TUTORIAL 8
STA437 WINTER 2015

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Contents

1. Tests Comparing Covariance Matrices

1.1. **Univariate Tests of Equality of Variances.** From univariate statistics we know the following test for equality of variances. Let s_1 and s_2 denote the standard deviations of a variable in two independent samples of size n_1 and n_2, respectively. If the null hypothesis of equality of the two population variances holds, the ration

$$F = \frac{s_1^2}{s_2^2}$$

deviates from unity only by sampling error. Under Normality assumptions, the distribution of the F-ratio is F with $n_1 - 1$ degrees of freedom and $n_2 - 1$ degrees of freedom. The hypothesis of equality is rejected if $F < f_l$ or $F > f_u$, where f_l and f_u are the lower and upper $\alpha/2$-quantiles of the null distribution. We are now going to indicate how this univariate F-ratio can be generalized to the multivariate case. There is a simple argument that allows us to define F-ratios also in the multivariate case: if the null hypothesis of equality of the two covariance matrices holds true, then the variance of every linear combination must be identical in both groups; conversely, if for every linear combination the variances are identical in both groups, then the covariance matrices must be the same. Our null hypothesis is therefore equivalent to the condition that for every linear combination the same standard deviation results in both groups, or, in other words, that the ratio of variances of every linear combination equals unity. This condition can be checked by determining linear combinations for which the empirical ratio of variances deviates as strongly as possible from one. Formally, we look at linear combinations

$$Y = a_1X_1 + a_2X_2 + ... + a_pX_p,$$

compute their variances s_1^2 and s_2^2 in both groups and form the ratio

$$F = F(a_1, a_2, ..., a_p) = \frac{s_1^2}{s_2^2}.$$

The particular linear combination for which F becomes maximal is called Y_{max}; analogously, Y_{min} will be the linear combination with minimal ratio of variances. If, instead of F, we form the reciprocal ratio

$$F' = \frac{1}{F} = \frac{s_2^2}{s_1^2}$$

then we obtain only the linear combinations $Y'_{max} = Y_{min}$ and $Y'_{min} = Y_{max}$.

The respective maximal and minimal ratios of variances are $F'_{max} = 1/F_{min}$ and $F'_{min} = 1/F_{max}$.

From this we see that the choice of group identification, i. e. which group is labelled first and which second, leads only to unessential changes in the results.

1.2. **Multivariate Tests of Equality of Variances.** In order to determine Y_{max} and Y_{min} in the case of $p \geq 2$ variables, one has to compute the p eigenvectors and associated eigenvalues of the so-called multivariate F-matrix (We provide the mathematical details in the appendix).

Each of the p eigenvectors contains the coefficients of a particular linear combination, and the associated eigenvalue gives just the corresponding value of the ratio of variances. We denote the eigenvalues by $\lambda_1, \lambda_2, ..., \lambda_p$ and order them decreasingly, that is

$$\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_p$$

With this notation we have $F_{max} = \lambda_1$ and $F_{min} = \lambda_p$. From now on we will devote our attention mainly to λ_1 and λ_p. The extreme eigenvalues F_{max} and F_{min} can be used to test the hypothesis of equality of the two covariance matrices as follows. Under the null hypothesis, these two extremes differ from 1 only by sampling error, and so λ_1 and λ_p would be expected close to 1. On the other hand, if F_{max} is much larger than 1 or F_{min} much smaller than 1, this indicates that the covariance structures on the two groups are not identical. The test statistic is thus actually a pair of statistics (λ_1, λ_p) or (F_{max}, F_{min}), and it is most often referred to as Roy's largest and smallest roots criterion. Since F_{max} and F_{min} are the maximum and minimum respectively over all linear combinations of p variables, they cannot be compared with critical values of the familiar F-distribution. What we need instead are quantiles of the so-called multivariate F-distribution; see table at the end. This table gives selected critical values c_{min} and c_{max} such that, under the null hypothesis of equality of both covariance matrices,

$$P(F_{min} \leq c_{min}) = 0.025$$

and

$$P(F_{max} \geq c_{max}) = 0.025.$$

At a significance level of approximately $\alpha = 5\%$, the null hypothesis is rejected if F_{min} is smaller than c_{min}, **or** if F_{max} is larger than c_{max}.

Example. Comparison of the Covariance Matrices of Genuine and Forged Bank Notes

This set of data comes from an inquiry that was conducted into genuine and forged thousand franc bills. For each attribute we introduce the following notation.

X_1: length of bill = LENGTH.
X_2: width of bill, measured on the left = LEFT.
X_3: width of bill, measured on the right = RIGHT.
X_4: width of margin at the bottom = BOTTOM.
X_5: width of margin at the top = TOP.
X_6: length of the image diagonal = DIAGONAL.

All measurements are given in millimetres. Below we show the covariance matrices for all six variables in both groups.

Covariance matrix of 100 genuine bills.

	length	left	right	bottom	top	diagonal
length	0.1502	0.0580	0.0573	0.0571	0.0145	0.0055
left	0.0580	0.1326	0.0859	0.0567	0.0491	-0.0431
right	0.0573	0.0859	0.1263	0.0582	0.0306	-0.0238
bottom	0.0571	0.0567	0.0582	0.4132	-0.2635	-0.0002
top	0.0145	0.0491	0.0306	-0.2635	0.4212	-0.0753
diagonal	0.0055	-0.0431	-0.0238	-0.0002	-0.0753	0.1998

Covariance matrix of 100 forged bills.

	length	left	right	bottom	top	diagonal
length	0.1240	0.0315	0.0240	-0.1006	0.0194	0.0116
left	0.0315	0.0650	0.0468	-0.0240	-0.0119	-0.0050
right	0.0240	0.0468	0.0889	-0.0186	0.0001	0.0342
bottom	-0.1006	-0.0240	-0.0186	1.2813	-0.4902	0.2385
top	0.0194	-0.0119	0.0001	-0.4902	0.4045	-0.0221
diagonal	0.0116	-0.0050	0.0342	0.2385	-0.0221	0.3112

In this example, maximization and minimization of the ratio s_F^2/s_G^2 yields the eigenvalues

$$F_{max} = 6.223,\ 1.675,\ 1.052,\ 0.900,\ 0.546,\ 0.284 = F_{min}.$$

From the tables of the multivariate F-distribution we obtain the critical values $c_{min} = 0.43$ and $c_{max} = 2.32$. Since $F_{min} < c_{min}$ as well as $F_{max} > c_{max}$, we conclude (at a significance level of approximately 5%) that the two covariance matrices are different.

R code

```
## genuine

g1<-c(0.1502,0.0580,0.0573,0.0571,0.0145,0.0055)

g2<-c(0,0.1326,0.0859,0.0567,0.0491,-0.0431)

g3<-c(0,0,0.1263,0.0582,0.0306,-0.0238)

g4<-c(0,0,0,0.4132,-0.2635,-0.0002)

g5<-c(0,0,0,0,0.4212,-0.0753)

g6<-c(0,0,0,0,0,0.1998)

SG<-cbind(g1,g2,g3,g4,g5,g6)

NEW.SG<-SG+t(SG)-diag(diag(SG),6,6)

NEW.SG

## forged

f1<-c(0.1240,0.0315,0.0240,-0.1006,0.0194,0.0116)

f2<-c(0,0.0650,0.0468,-0.0240,-0.0119,-0.0050)
```

```
f3<-c(0,0,0.0889,-0.0186,0.0001,0.0342)

f4<-c(0,0,0,1.2813,-0.4902,0.2385)

f5<-c(0,0,0,0,0.4045,-0.0221)

f6<-c(0,0,0,0,0,0.3112)

SF<-cbind(f1,f2,f3,f4,f5,f6)

NEW.SF<-SF+t(SF)-diag(diag(SF),6,6)

NEW.SF


## FINDING EIGENVALUES OF PRODUCT
## PROD 1 = (NEW.SG)^{-1} NEW.SF

prod1<-solve(NEW.SG)%*%NEW.SF

eigen(prod1)


## FINDING EIGENVALUES OF PRODUCT
## PROD 2 = (NEW.SF)^{-1} NEW.SG

prod2<-solve(NEW.SF)%*%NEW.SG

eigen(prod2)
```

1.3. **Testing the Equality of Several Covariance Matrices.** The hypothesis

$$H_0 : \mathbf{\Sigma_1} = \mathbf{\Sigma_2} = ... = \mathbf{\Sigma_k}$$

of the equality of the covariance matrices of k p-dimensional Multinormal populations can be tested against the alternative of general positive definite matrices by a modified generalized likelihood-ratio statistic. Let $\mathbf{S_i}$ be the unbiased estimate of $\mathbf{\Sigma_i}$ based on ν_i degrees of freedom, where $\nu_i = n_i - 1$ for the usual case of a random sample of n_i observation vectors from the ith population. When H_0 is true

$$\mathbf{S} = \frac{1}{\sum \nu_i} \sum_{i=1}^{k} \nu_i \mathbf{S}_i$$

is the pooled estimate of the common covariance matrix. The test statistic is

$$M = \sum_{i=1}^{k} \nu_i ln|\mathbf{S}| - \sum_{i=1}^{k} \nu_i ln|\mathbf{S}_i|$$

it has been shown that if the scale factor

$$C^{-1} = 1 - \frac{2p^2 + 3p - 1}{6(p+1)(k-1)} \left(\sum_{i=1}^{k} \frac{1}{\nu_i} - \frac{1}{\sum_{i=1}^{k} \nu_i} \right)$$

is introduced the quantity MC^{-1} is approximately distributed as a chi-squared variate with degrees of freedom $\frac{1}{2}(k-1)p(p+1)$ as the ν_i become larger. If all the ν_i are equal to n,

$$C^{-1} = 1 - \frac{(2p^2 + 3p - 1)(k+1)}{6(p+1)(kn)}$$

Example. In a reaction-time study 32 male and 32 female young normal subjects reacted to visual stimuli preceded by warning intervals of different lengths. The sample covariance matrices of reaction times with preparatory intervals of 0.5 and 15 sec were

$$\mathbf{S}_M = \begin{pmatrix} 4.32 & 1.88 \\ 1.88 & 9.18 \end{pmatrix},$$

$$\mathbf{S}_F = \begin{pmatrix} 2.52 & 1.90 \\ 1.90 & 10.06 \end{pmatrix},$$

where the elements are in units of 10^{-4} sec^2. It is desired to test the hypothesis of a common covariance matrix in both sexes. Use $\alpha = 0.05$.

Solution

$p = 2$, $n_1 = n_2 = 32$, $\nu_1 = \nu_2 = 31$

$$\mathbf{S} = \frac{31}{62}\mathbf{S}_1 + \frac{31}{62}\mathbf{S}_2 = \begin{pmatrix} 3.42 & 1.89 \\ 1.89 & 9.62 \end{pmatrix},$$

$$M = 62ln(29.328) - 31[ln(36.123) + ln(21.741)] = 2.82$$

$C^{-1} = 0.965$, and since $MC^{-1} = 2.72$ is much smaller than the percentage point $\chi^2_{0.05,3} = 7.81$, we conclude that the null hypothesis is indeed tenable.

Exercise Test the hypothesis $H_0 : \mathbf{\Sigma_1} = \mathbf{\Sigma_2}$ for the psychological data.

1.4. **Independence of Two Subvectors.** Suppose the observation vector is partitioned into two subvectors of interest, which we label $\mathbf{y}$ and $\mathbf{x}$, where $\mathbf{y}$ is $p \times 1$ and $\mathbf{x}$ is $q \times 1$. The corresponding partitioning of the population covariance matrix is

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{yx} \\ \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{xx} \end{pmatrix},$$

with analogous partitioning of $\mathbf{S}$ and $\mathbf{R}$

$$\mathbf{S} = \begin{pmatrix} \mathbf{S}_{yy} & \mathbf{S}_{yx} \\ \mathbf{S}_{xy} & \mathbf{S}_{xx} \end{pmatrix},$$

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_{yy} & \mathbf{R}_{yx} \\ \mathbf{R}_{xy} & \mathbf{R}_{xx} \end{pmatrix},$$

The hypothesis of independence of $\mathbf{y}$ and $\mathbf{x}$ can be expressed as

$$H_0 : \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{yy} & \mathbf{O} \\ \mathbf{O} & \boldsymbol{\Sigma}_{xx} \end{pmatrix},$$

or $H_0 : \boldsymbol{\Sigma}_{yx} = \mathbf{O}$.

The likelihood ratio test statistic for $H_0 : \boldsymbol{\Sigma}_{yx} = \mathbf{O}$ is given by

$$\Lambda = \frac{|\mathbf{S}|}{|\mathbf{S}_{yy}||\mathbf{S}_{xx}|} = \frac{|\mathbf{R}|}{|\mathbf{R}_{yy}||\mathbf{R}_{xx}|}$$

which is distributed as $\Lambda_{p,q,n-1-q}$. We reject H_0 if $\Lambda \leq \Lambda_\alpha$. Critical values for Wilk's Λ are given in Table A.9 using $\nu_H = q$ and $\nu_E = n - 1 - q$.

Example In an investigation of the relation of the Wechsler Adult Intelligence Scale to age. Researchers obtained this matrix of correlations among the digit span and vocabulary subsets, chronological age, and years of formal education:

$$\mathbf{R} = \begin{pmatrix} 1 & 0.45 & -0.19 & 0.43 \\ 0.45 & 1 & -0.02 & 0.62 \\ -0.19 & -0.02 & 1 & -0.29 \\ 0.43 & 0.62 & -0.29 & 1 \end{pmatrix},$$

The sample consisted of $N = 933$ men and women aged 25 to 64. From these data we wish to test at level $\alpha = 0.05$ the hypothesis that the pair of WAIS subtest variates is distributed independently of the age and education variates.

Solution

$p = q = 2$, $\nu_H = 2$, and $\nu_E = 933 - 1 - 2 = 930$

$|\mathbf{R}| = 0.4015025$

$|\mathbf{R}_{xx}| = 0.7975$

$|\mathbf{R}_{yy}| = 0.9159$

$$\Lambda = \frac{|\mathbf{R}|}{|\mathbf{R}_{yy}||\mathbf{R}_{xx}|} = \frac{0.4015025}{(0.7975)(0.9159)} = 0.5497$$

$$\Lambda_{0.05,2,2,930} \approx 0.9955$$

Since $\Lambda = 0.5497 < \Lambda_{0.05,2,2,930} \approx 0.9955$, we reject the hypothesis of independence. We must conclude that the subtests are dependent upon age and education.

Exercise Test independence of (y_1, y_2) and (x_1, x_2) for the sons data (sons.dat).

2. Appendix

Let now S_1 and S_2 denote the $p \times p$ covariance matrices of two samples. To find the linear combinations with extreme variance ratios, we form the ratio

$$\frac{\mathbf{a}'\mathbf{S_2}\mathbf{a}}{\mathbf{a}'\mathbf{S_1}\mathbf{a}} = \frac{\mathbf{a}'\mathbf{S_1^{1/2}}\mathbf{S_1^{-1/2}}\mathbf{S_2}\mathbf{S_1^{-1/2}}\mathbf{S_1^{1/2}}\mathbf{a}}{\mathbf{a}'\mathbf{S_1^{1/2}}\mathbf{S_1^{1/2}}\mathbf{a}}$$

Let $\mathbf{x} = \mathbf{S_1^{1/2}}\mathbf{a}$ and recall that $(\mathbf{S_1^{1/2}})' = \mathbf{S_1^{1/2}}$, then

$$\max_{\mathbf{a}} \frac{\mathbf{a}'\mathbf{S_2}\mathbf{a}}{\mathbf{a}'\mathbf{S_1}\mathbf{a}} = \max_{\mathbf{x}} \frac{\mathbf{x}'\mathbf{S_1^{-1/2}}\mathbf{S_2}\mathbf{S_1^{-1/2}}\mathbf{x}}{\mathbf{x}'\mathbf{x}}$$

Using our result for maximization of quadratic forms from tutorial 4, we have

$$\max_{\mathbf{x}} \frac{\mathbf{x}'\mathbf{S_1^{-1/2}}\mathbf{S_2}\mathbf{S_1^{-1/2}}\mathbf{x}}{\mathbf{x}'\mathbf{x}} = \lambda_1$$

where λ_1 is the largest eigenvalue of $\mathbf{S_1^{-1/2}}\mathbf{S_2}\mathbf{S_1^{-1/2}}$. Now, using the definition of similar matrices and the fact that similar matrices have the same eigenvalues, we can show that λ_1 is also the largest eigenvalue of $\mathbf{S_1^{-1}}\mathbf{S_2}$ (again, see tutorial 4).

Table 11.3 Upper 2.5% quantiles of the largest characteristic root of the multivariate F-matrix

P=2

ν_2/ν_1	43	53	63	73	83	103	123	143	173	203	243	283	343	403	603
43	2.245	2.188	2.147	2.117	2.094	2.060	2.036	2.019	2.001	1.988	1.975	1.966	1.956	1.949	1.936
53	2.124	2.065	2.023	1.992	1.968	1.933	1.908	1.891	1.871	1.857	1.844	1.834	1.824	1.817	1.803
63	2.043	1.983	1.940	1.908	1.884	1.847	1.822	1.804	1.784	1.769	1.755	1.745	1.734	1.727	1.712
73	1.985	1.924	1.881	1.848	1.823	1.786	1.760	1.741	1.720	1.705	1.691	1.680	1.669	1.661	1.645
83	1.942	1.880	1.836	1.803	1.777	1.739	1.713	1.693	1.672	1.656	1.642	1.631	1.619	1.611	1.594
103	1.881	1.818	1.773	1.739	1.713	1.673	1.646	1.626	1.603	1.587	1.571	1.560	1.547	1.538	1.521
123	1.841	1.777	1.731	1.696	1.669	1.629	1.601	1.579	1.556	1.539	1.523	1.511	1.498	1.488	1.470
143	1.812	1.747	1.701	1.666	1.638	1.597	1.568	1.546	1.522	1.505	1.488	1.475	1.461	1.452	1.433
173	1.781	1.716	1.689	1.633	1.604	1.562	1.532	1.510	1.485	1.467	1.449	1.436	1.422	1.412	1.391
203	1.759	1.694	1.646	1.609	1.581	1.538	1.507	1.484	1.459	1.440	1.422	1.408	1.393	1.383	1.361
243	1.739	1.673	1.624	1.587	1.558	1.515	1.483	1.460	1.434	1.414	1.395	1.381	1.366	1.354	1.332
283	1.724	1.657	1.609	1.572	1.542	1.498	1.468	1.442	1.415	1.396	1.376	1.361	1.345	1.334	1.310
343	1.709	1.642	1.592	1.555	1.525	1.480	1.448	1.423	1.396	1.375	1.355	1.340	1.323	1.311	1.287
403	1.698	1.630	1.581	1.543	1.513	1.467	1.434	1.410	1.382	1.361	1.340	1.325	1.307	1.295	1.269
603	1.677	1.609	1.559	1.520	1.489	1.443	1.409	1.384	1.355	1.333	1.311	1.295	1.276	1.263	1.235

P=3

ν_2/ν_1	44	54	64	74	84	104	124	144	174	204	244	284	344	404	604
44	2.588	2.516	2.465	2.427	2.398	2.356	2.326	2.305	2.281	2.265	2.248	2.237	2.224	2.216	2.199
54	2.418	2.345	2.293	2.255	2.225	2.181	2.151	2.128	2.104	2.087	2.070	2.058	2.045	2.036	2.018
64	2.306	2.232	2.180	2.140	2.110	2.065	2.034	2.011	1.986	1.968	1.950	1.938	1.924	1.915	1.896
74	2.226	2.152	2.099	2.059	2.028	1.982	1.950	1.927	1.901	1.882	1.864	1.851	1.837	1.827	1.808
84	2.167	2.092	2.038	1.998	1.966	1.920	1.887	1.863	1.837	1.818	1.799	1.786	1.771	1.761	1.741
104	2.084	2.008	1.954	1.912	1.880	1.832	1.798	1.773	1.746	1.726	1.707	1.692	1.677	1.686	1.645
124	2.030	1.953	1.897	1.855	1.822	1.773	1.739	1.713	1.684	1.664	1.644	1.629	1.613	1.601	1.579
144	1.990	1.913	1.857	1.814	1.781	1.731	1.696	1.669	1.640	1.619	1.598	1.583	1.566	1.554	1.531
174	1.949	1.871	1.814	1.771	1.737	1.686	1.650	1.622	1.592	1.570	1.549	1.533	1.515	1.503	1.478
204	1.920	1.841	1.784	1.740	1.706	1.654	1.617	1.589	1.558	1.536	1.514	1.497	1.479	1.466	1.440
244	1.893	1.814	1.756	1.711	1.676	1.624	1.586	1.558	1.526	1.503	1.480	1.462	1.444	1.430	1.403
284	1.873	1.794	1.735	1.691	1.655	1.602	1.564	1.535	1.503	1.479	1.455	1.437	1.418	1.404	1.376
344	1.853	1.773	1.714	1.689	1.633	1.579	1.540	1.510	1.477	1.453	1.428	1.410	1.390	1.375	1.346
404	1.838	1.758	1.699	1.653	1.617	1.563	1.523	1.493	1.459	1.434	1.410	1.391	1.370	1.355	1.324
604	1.811	1.730	1.670	1.624	1.587	1.531	1.491	1.460	1.425	1.399	1.373	1.353	1.331	1.315	1.282

P=4

ν_2/ν_1	45	55	65	75	85	105	125	145	175	205	245	285	345	405	605
45	2.908	2.825	2.765	2.721	2.686	2.636	2.601	2.575	2.547	2.527	2.508	2.494	2.479	2.468	2.448
55	2.689	2.605	2.545	2.499	2.464	2.413	2.377	2.351	2.322	2.301	2.281	2.267	2.251	2.240	2.219
65	2.545	2.460	2.400	2.354	2.318	2.266	2.229	2.202	2.173	2.152	2.131	2.116	2.100	2.089	2.067
75	2.444	2.359	2.297	2.251	2.215	2.162	2.124	2.097	2.067	2.045	2.024	2.008	1.992	1.980	1.957
85	2.369	2.283	2.221	2.174	2.138	2.084	2.046	2.018	1.987	1.965	1.943	1.927	1.910	1.898	1.875
105	2.264	2.178	2.115	2.067	2.030	1.974	1.935	1.906	1.875	1.851	1.829	1.812	1.795	1.782	1.757
125	2.195	2.108	2.044	1.996	1.958	1.902	1.862	1.832	1.799	1.775	1.752	1.735	1.716	1.703	1.677
145	2.146	2.058	1.994	1.945	1.907	1.850	1.809	1.779	1.745	1.720	1.696	1.679	1.659	1.646	1.619
175	2.094	2.006	1.941	1.892	1.852	1.794	1.753	1.721	1.687	1.661	1.637	1.618	1.598	1.584	1.555
205	2.058	1.969	1.904	1.854	1.814	1.755	1.718	1.681	1.646	1.620	1.594	1.575	1.554	1.539	1.510
245	2.024	1.935	1.869	1.818	1.778	1.718	1.675	1.643	1.606	1.580	1.553	1.534	1.512	1.496	1.465
285	2.000	1.910	1.844	1.793	1.753	1.692	1.648	1.615	1.578	1.551	1.524	1.504	1.481	1.465	1.433
345	1.974	1.884	1.817	1.766	1.725	1.664	1.619	1.586	1.548	1.520	1.492	1.471	1.448	1.431	1.398
405	1.957	1.866	1.799	1.747	1.706	1.644	1.599	1.565	1.526	1.498	1.469	1.448	1.424	1.407	1.372
605	1.923	1.831	1.764	1.711	1.669	1.606	1.560	1.525	1.485	1.456	1.426	1.403	1.378	1.359	1.322

P=5

ν_2/ν_2	46	56	66	76	86	106	126	146	176	206	246	286	346	406	606
46	3.220	3.125	3.058	3.007	2.967	2.909	2.869	2.839	2.807	2.784	2.762	2.746	2.728	2.716	2.693
56	2.949	2.854	2.786	2.735	2.695	2.636	2.596	2.565	2.533	2.509	2.486	2.470	2.452	2.439	2.415
66	2.772	2.678	2.609	2.558	2.517	2.458	2.417	2.386	2.352	2.328	2.305	2.288	2.269	2.256	2.231
76	2.649	2.554	2.485	2.433	2.392	2.332	2.290	2.259	2.224	2.200	2.176	2.158	2.139	2.126	2.100
86	2.557	2.462	2.393	2.340	2.299	2.238	2.196	2.164	2.129	2.104	2.079	2.061	2.042	2.028	2.002
106	2.430	2.334	2.265	2.213	2.170	2.108	2.064	2.031	1.995	1.969	1.944	1.925	1.905	1.891	1.863
126	2.347	2.250	2.180	2.126	2.084	2.021	1.976	1.943	1.906	1.879	1.853	1.834	1.813	1.798	1.768
146	2.288	2.191	2.120	2.066	2.023	1.959	1.914	1.880	1.842	1.815	1.788	1.768	1.746	1.731	1.700
176	2.226	2.128	2.057	2.002	1.959	1.894	1.847	1.813	1.774	1.745	1.718	1.697	1.674	1.658	1.626
206	2.182	2.084	2.013	1.957	1.913	1.848	1.801	1.765	1.726	1.697	1.668	1.647	1.624	1.607	1.574
246	2.142	2.043	1.971	1.915	1.871	1.804	1.756	1.720	1.680	1.650	1.621	1.599	1.575	1.557	1.522
286	2.113	2.014	1.941	1.885	1.840	1.773	1.725	1.688	1.647	1.617	1.586	1.564	1.539	1.521	1.485
346	2.082	1.983	1.910	1.853	1.808	1.740	1.691	1.654	1.612	1.581	1.550	1.526	1.501	1.482	1.444
406	2.061	1.961	1.888	1.831	1.786	1.717	1.667	1.629	1.587	1.555	1.524	1.500	1.473	1.454	1.415
606	2.021	1.921	1.846	1.789	1.743	1.673	1.622	1.583	1.539	1.506	1.473	1.448	1.420	1.400	1.358

Table 11.3 (Continued)

P=6

ν_2/ν_1	47	57	67	77	87	107	127	147	177	207	247	287	347	407	607
47	3.527	3.422	3.347	3.290	3.246	3.181	3.136	3.102	3.066	3.040	3.015	2.996	2.977	2.963	2.936
57	3.202	3.098	3.023	2.966	2.922	2.856	2.811	2.777	2.740	2.714	2.688	2.669	2.649	2.635	2.607
67	2.992	2.888	2.813	2.756	2.711	2.646	2.599	2.565	2.528	2.501	2.474	2.455	2.435	2.420	2.392
77	2.845	2.741	2.666	2.609	2.564	2.497	2.451	2.416	2.378	2.350	2.324	2.304	2.283	2.268	2.239
87	2.737	2.633	2.557	2.500	2.455	2.388	2.340	2.305	2.266	2.238	2.211	2.191	2.170	2.154	2.125
107	2.587	2.483	2.407	2.349	2.303	2.235	2.187	2.151	2.111	2.082	2.054	2.034	2.011	1.995	1.964
127	2.489	2.385	2.308	2.250	2.204	2.134	2.085	2.049	2.008	1.978	1.949	1.928	1.905	1.888	1.856
147	2.420	2.315	2.238	2.179	2.133	2.063	2.013	1.976	1.934	1.904	1.874	1.852	1.829	1.811	1.778
177	2.347	2.242	2.165	2.105	2.058	1.987	1.937	1.898	1.856	1.825	1.794	1.772	1.747	1.729	1.694
207	2.297	2.191	2.114	2.054	2.006	1.934	1.883	1.844	1.801	1.769	1.738	1.714	1.689	1.670	1.634
247	2.249	2.144	2.065	2.005	1.957	1.884	1.832	1.793	1.749	1.716	1.684	1.660	1.633	1.614	1.576
287	2.216	2.110	2.031	1.970	1.922	1.849	1.796	1.756	1.711	1.678	1.645	1.620	1.593	1.573	1.534
347	2.180	2.074	1.995	1.934	1.885	1.811	1.757	1.717	1.671	1.637	1.603	1.578	1.550	1.529	1.488
407	2.155	2.049	1.969	1.908	1.859	1.784	1.730	1.689	1.643	1.608	1.574	1.548	1.519	1.498	1.455
607	2.109	2.002	1.922	1.860	1.810	1.734	1.679	1.637	1.589	1.553	1.517	1.490	1.459	1.437	1.391

P=7

ν_2/ν_1	48	58	68	78	88	108	128	148	178	208	248	288	348	408	608
48	3.833	3.719	3.636	3.574	3.525	3.453	3.407	3.366	3.326	3.297	3.268	3.248	3.226	3.210	3.180
58	3.451	3.339	3.257	3.195	3.147	3.075	3.025	2.987	2.946	2.917	2.889	2.868	2.845	2.830	2.799
68	3.206	3.095	3.013	2.951	2.902	2.830	2.780	2.742	2.701	2.671	2.642	2.621	2.598	2.582	2.551
78	3.036	2.924	2.843	2.781	2.732	2.659	2.608	2.570	2.529	2.498	2.469	2.447	2.424	2.408	2.376
88	2.910	2.799	2.717	2.655	2.606	2.533	2.482	2.443	2.401	2.370	2.340	2.318	2.295	2.278	2.245
108	2.738	2.626	2.545	2.482	2.432	2.359	2.306	2.267	2.224	2.192	2.162	2.139	2.114	2.097	2.063
128	2.625	2.513	2.431	2.368	2.318	2.244	2.191	2.151	2.107	2.074	2.043	2.020	1.994	1.976	1.941
148	2.545	2.434	2.351	2.288	2.238	2.162	2.108	2.068	2.023	1.990	1.958	1.934	1.908	1.889	1.853
178	2.462	2.350	2.267	2.203	2.153	2.076	2.022	1.981	1.935	1.901	1.868	1.843	1.816	1.797	1.758
208	2.404	2.292	2.209	2.145	2.094	2.017	1.961	1.919	1.873	1.838	1.804	1.779	1.751	1.731	1.691
248	2.350	2.237	2.154	2.090	2.038	1.960	1.904	1.861	1.814	1.779	1.744	1.718	1.689	1.668	1.627
288	2.311	2.199	2.115	2.050	1.998	1.920	1.863	1.820	1.772	1.736	1.700	1.674	1.644	1.623	1.589
348	2.271	2.158	2.074	2.009	1.956	1.877	1.820	1.776	1.727	1.690	1.654	1.626	1.596	1.574	1.529
408	2.243	2.129	2.045	1.980	1.927	1.847	1.790	1.745	1.695	1.658	1.621	1.593	1.562	1.539	1.493
608	2.190	2.076	1.991	1.925	1.872	1.791	1.732	1.687	1.635	1.597	1.558	1.529	1.496	1.472	1.422

P=8

ν_2/ν_1	49	59	69	79	89	109	129	149	179	209	249	289	349	409	609
49	4.139	4.016	3.927	3.859	3.806	3.728	3.673	3.632	3.588	3.555	3.524	3.502	3.477	3.460	3.427
59	3.699	3.579	3.491	3.424	3.371	3.293	3.238	3.198	3.153	3.121	3.090	3.067	3.042	3.025	2.991
69	3.418	3.298	3.211	3.145	3.092	3.014	2.959	2.918	2.873	2.841	2.809	2.786	2.761	2.743	2.709
79	3.223	3.104	3.017	2.950	2.898	2.820	2.764	2.723	2.678	2.645	2.613	2.590	2.564	2.546	2.511
89	3.079	2.961	2.874	2.808	2.755	2.676	2.621	2.579	2.533	2.500	2.468	2.444	2.418	2.400	2.364
109	2.883	2.765	2.678	2.611	2.558	2.479	2.423	2.381	2.334	2.300	2.267	2.242	2.216	2.197	2.160
129	2.755	2.637	2.550	2.483	2.430	2.350	2.293	2.250	2.203	2.168	2.134	2.109	2.081	2.062	2.023
149	2.665	2.547	2.460	2.393	2.339	2.258	2.201	2.157	2.109	2.074	2.039	2.013	1.985	1.965	1.926
179	2.571	2.453	2.365	2.298	2.244	2.162	2.104	2.060	2.011	1.974	1.939	1.912	1.883	1.862	1.821
209	2.505	2.387	2.300	2.232	2.177	2.095	2.036	1.992	1.942	1.905	1.868	1.841	1.811	1.790	1.747
249	2.444	2.326	2.238	2.170	2.115	2.033	1.973	1.927	1.876	1.839	1.801	1.773	1.743	1.720	1.676
289	2.401	2.283	2.195	2.126	2.071	1.988	1.927	1.881	1.830	1.791	1.753	1.725	1.693	1.670	1.625
349	2.355	2.237	2.149	2.080	2.024	1.940	1.879	1.833	1.780	1.741	1.702	1.673	1.640	1.617	1.569
409	2.324	2.205	2.117	2.047	1.992	1.907	1.846	1.799	1.745	1.706	1.666	1.636	1.603	1.578	1.529
609	2.264	2.145	2.056	1.987	1.930	1.845	1.782	1.734	1.679	1.638	1.597	1.566	1.531	1.505	1.452

P=9

ν_2/ν_1	50	60	70	80	90	110	130	150	180	210	250	290	350	410	610
50	4.447	4.316	4.220	4.147	4.090	4.005	3.945	3.901	3.852	3.817	3.784	3.759	3.732	3.713	3.676
60	3.946	3.818	3.724	3.653	3.596	3.512	3.453	3.409	3.361	3.326	3.292	3.267	3.240	3.221	3.184
70	3.628	3.501	3.408	3.337	3.281	3.197	3.138	3.094	3.045	3.010	2.976	2.951	2.924	2.905	2.867
80	3.407	3.281	3.189	3.118	3.062	2.979	2.919	2.875	2.826	2.791	2.756	2.731	2.704	2.684	2.646
90	3.245	3.121	3.029	2.958	2.902	2.818	2.759	2.714	2.665	2.629	2.594	2.568	2.541	2.521	2.482
110	3.025	2.901	2.809	2.738	2.682	2.598	2.538	2.493	2.443	2.406	2.371	2.344	2.316	2.295	2.255
130	2.881	2.757	2.666	2.595	2.539	2.454	2.393	2.347	2.297	2.260	2.223	2.196	2.167	2.146	2.105
150	2.780	2.657	2.565	2.494	2.437	2.352	2.291	2.245	2.193	2.156	2.119	2.091	2.061	2.039	1.997
180	2.675	2.552	2.460	2.389	2.332	2.246	2.184	2.137	2.085	2.046	2.008	1.980	1.949	1.926	1.882
210	2.692	2.479	2.387	2.318	2.258	2.172	2.109	2.062	2.009	1.969	1.930	1.902	1.870	1.847	1.801
250	2.534	2.411	2.319	2.247	2.189	2.102	2.039	1.991	1.937	1.897	1.857	1.827	1.795	1.771	1.724
290	2.486	2.363	2.271	2.199	2.141	2.053	1.989	1.940	1.886	1.845	1.804	1.774	1.741	1.716	1.668
350	2.435	2.312	2.220	2.147	2.089	2.001	1.936	1.887	1.831	1.790	1.748	1.717	1.683	1.658	1.607
410	2.400	2.277	2.184	2.112	2.053	1.964	1.899	1.850	1.793	1.751	1.709	1.677	1.642	1.616	1.564
610	2.334	2.211	2.118	2.045	1.986	1.896	1.830	1.779	1.721	1.678	1.634	1.601	1.564	1.536	1.480

Table A.9. Lower Critical Values of Wilks Λ, $\alpha = .05$

$$\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{H}|} = \prod_{i=1}^{s} \frac{1}{1+\lambda_i},$$

where $\lambda_1, \lambda_2, \ldots, \lambda_s$ are eigenvalues of $\mathbf{E}^{-1}\mathbf{H}$. Reject H_0 if $\Lambda \leq$ table value. [a] Multiply entry by 10^{-3}.

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 1$						
1	6.16[a]	2.50[a]	1.54[a]	1.11[a]	.868[a]	.712[a]	.603[a]	.523[a]	.462[a]	.413[a]	.374[a]	.341[a]
2	.098	.050	.034	.025	.020	.017	.015	.013	.011	.010	9.28[a]	8.51[a]
3	.229	.136	.097	.076	.062	.053	.046	.041	.036	.033	.030	.028
4	.342	.224	.168	.135	.113	.098	.086	.076	.069	.063	.058	.053
5	.431	.302	.236	.194	.165	.144	.128	.115	.104	.096	.088	.082
6	.501	.368	.296	.249	.215	.189	.169	.153	.140	.129	.119	.111
7	.556	.425	.349	.298	.261	.232	.209	.190	.175	.161	.150	.140
8	.601	.473	.396	.343	.303	.271	.246	.225	.208	.193	.180	.169
9	.638	.514	.437	.382	.341	.308	.281	.258	.239	.223	.209	.196
10	.668	.549	.473	.418	.376	.341	.313	.289	.269	.251	.236	.222
11	.694	.580	.505	.450	.407	.372	.343	.318	.297	.278	.262	.247
12	.717	.607	.534	.479	.436	.400	.370	.345	.323	.304	.286	.271
13	.736	.631	.560	.506	.462	.426	.396	.370	.347	.327	.310	.294
14	.753	.652	.583	.529	.486	.450	.420	.393	.370	.350	.332	.315
15	.768	.671	.603	.551	.508	.473	.442	.415	.392	.371	.352	.336
16	.781	.688	.622	.571	.529	.493	.462	.436	.412	.391	.372	.355
17	.792	.703	.639	.589	.548	.512	.482	.455	.431	.410	.390	.373
18	.803	.717	.655	.606	.565	.530	.499	.473	.449	.427	.408	.390
19	.813	.730	.669	.621	.581	.546	.516	.490	.466	.444	.425	.407
20	.821	.741	.683	.636	.596	.562	.532	.505	.482	.460	.440	.423
21	.829	.752	.695	.649	.610	.576	.547	.520	.497	.475	.455	.437
22	.836	.762	.706	.661	.623	.590	.561	.534	.511	.489	.470	.452
23	.843	.771	.717	.673	.635	.603	.574	.548	.524	.503	.483	.465
24	.849	.779	.727	.684	.647	.615	.586	.560	.537	.516	.496	.478
25	.855	.787	.736	.694	.658	.626	.598	.572	.549	.528	.508	.490
26	.860	.794	.744	.703	.668	.637	.609	.583	.560	.539	.520	.502
27	.865	.801	.752	.712	.677	.647	.619	.594	.571	.551	.531	.513
28	.870	.807	.760	.721	.686	.656	.629	.604	.582	.561	.542	.524
29	.874	.813	.767	.729	.695	.665	.638	.614	.592	.571	.552	.535
30	.878	.819	.774	.736	.703	.674	.647	.623	.601	.581	.562	.544
40	.907	.861	.824	.793	.766	.741	.718	.696	.677	.658	.641	.625
60	.938	.905	.879	.856	.835	.816	.798	.781	.766	.751	.736	.723
80	.953	.928	.907	.889	.873	.858	.843	.829	.816	.804	.792	.780
100	.962	.942	.925	.910	.897	.884	.872	.860	.849	.838	.828	.818
120	.968	.951	.937	.925	.913	.902	.891	.882	.872	.863	.854	.845
140	.973	.958	.946	.935	.925	.915	.906	.897	.889	.881	.873	.865
170	.978	.965	.955	.946	.937	.929	.922	.914	.907	.900	.893	.887
200	.981	.970	.962	.954	.947	.940	.933	.926	.920	.914	.908	.902
240	.984	.975	.968	.961	.955	.949	.944	.938	.933	.928	.923	.918
320	.988	.981	.976	.971	.966	.962	.957	.953	.949	.945	.941	.937
440	.991	.986	.982	.979	.975	.972	.969	.966	.963	.960	.957	.954
600	.994	.990	.987	.984	.982	.979	.977	.975	.972	.970	.968	.966
800	.995	.993	.990	.988	.986	.984	.983	.981	.979	.977	.976	.974
1000	.996	.994	.992	.991	.989	.988	.986	.985	.983	.982	.981	.979

(continued)

Table A.9. (*Continued*)

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p = 2$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	2.50[a]	.641[a]	.287[a]	.162[a]	.104[a]	.072[a]	.053[a]	.041[a]	.032[a]	.026[a]	.022[a]	.018[a]
3	.050	.018	9.53[a]	5.84[a]	3.95[a]	2.85[a]	2.15[a]	1.68[a]	1.35[a]	1.11[a]	.928[a]	.787[a]
4	.136	.062	.036	.023	.017	.012	9.56[a]	7.62[a]	6.21[a]	5.17[a]	4.36[a]	3.73[a]
5	.224	.117	.074	.051	.037	.028	.023	.018	.015	.013	.011	.009
6	.302	.175	.116	.084	.063	.049	.040	.033	.027	.023	.020	.017
7	.368	.230	.160	.119	.092	.074	.060	.050	.042	.036	.032	.028
8	.4256	.280	.203	.155	.122	.099	.082	.069	.059	.051	.045	.040
9	.473	.326	.243	.190	.153	.126	.106	.090	.078	.068	.060	.053
10	.514	.367	.281	.223	.183	.152	.129	.111	.097	.085	.075	.067
11	.549	.404	.316	.255	.212	.179	.153	.133	.116	.102	.091	.082
12	.580	.437	.348	.286	.240	.204	.176	.154	.136	.120	.108	.097
13	.607	.467	.378	.314	.266	.229	.199	.175	.155	.138	.124	.112
14	.631	.495	.405	.340	.291	.252	.221	.195	.174	.156	.141	.128
15	.652	.519	.431	.365	.315	.275	.242	.215	.193	.174	.157	.143
16	.671	.542	.454	.389	.337	.296	.263	.235	.211	.191	.174	.159
17	.688	.562	.476	.410	.359	.317	.282	.254	.229	.208	.190	.174
18	.703	.581	.496	.431	.379	.337	.301	.272	.246	.225	.206	.189
19	.717	.598	.515	.450	.398	.355	.320	.289	.263	.241	.221	.204
20	.730	.614	.532	.468	.416	.373	.337	.306	.279	.256	.236	.218
21	.741	.629	.548	.485	.433	.390	.354	.322	.295	.271	.251	.232
22	.752	.643	.564	.501	.449	.406	.370	.338	.310	.286	.265	.246
23	.762	.656	.578	.516	.465	.422	.385	.353	.325	.300	.279	.259
24	.771	.668	.591	.530	.479	.436	.399	.367	.339	.314	.292	.272
25	.779	.679	.604	.544	.493	.450	.413	.381	.353	.328	.305	.285
26	.787	.689	.616	.556	.506	.464	.427	.395	.366	.341	.318	.297
27	.794	.699	.627	.568	.519	.477	.440	.407	.379	.353	.330	.309
28	.801	.708	.638	.580	.531	.489	.452	.420	.391	.365	.342	.321
29	.807	.717	.648	.591	.542	.501	.464	.432	.403	.377	.354	.332
30	.813	.725	.657	.601	.553	.512	.475	.443	.414	.388	.365	.344
40	.858	.786	.730	.682	.640	.602	.568	.537	.509	.484	.460	.439
60	.903	.853	.811	.774	.741	.710	.682	.656	.632	.609	.588	.568
80	.927	.888	.854	.825	.798	.772	.749	.727	.706	.686	.667	.649
100	.941	.909	.882	.857	.834	.813	.793	.774	.755	.738	.721	.705
120	.951	.924	.900	.879	.860	.841	.823	.807	.791	.775	.760	.746
140	.958	.934	.914	.895	.878	.862	.846	.831	.817	.803	.790	.777
170	.965	.946	.929	.913	.898	.885	.871	.859	.846	.834	.823	.812
200	.970	.954	.939	.926	.913	.901	.889	.878	.867	.857	.847	.837
240	.975	.961	.949	.938	.927	.917	.907	.897	.888	.879	.870	.862
320	.981	.971	.962	.953	.945	.937	.929	.922	.914	.907	.901	.894
440	.986	.979	.972	.965	.959	.953	.948	.942	.937	.932	.926	.921
600	.990	.984	.979	.975	.970	.966	.961	.957	.953	.949	.945	.942
800	.993	.988	.984	.981	.977	.974	.971	.968	.965	.962	.959	.956
1000	.994	.991	.987	.985	.982	.979	.977	.974	.972	.969	.967	.964

[a] Multiply entry by 10^{-3}.

(continued)

Table A.9. ***(Continued)***

	ν_H											
ν_E	1	2	3	4	5	6	7	8	9	10	11	12
						$p = 3$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.001[a]	.002[a]	.004[a]	.005[a]	.008[a]	.010[a]	.013[a]
3	1.70[a]	.354[a]	.179[a]	.127[a]	.105[a]	.095[a]	.091[a]	.090[a]	.091[a]	.092[a]	.095[a]	.098[a]
4	.034	.010	.004	.002	.001	.001	.809[a]	.659[a]	.562[a]	.496[a]	.449[a]	.416[a]
5	.097	.036	.018	.010	6.36[a]	4.37[a]	3.20[a]	2.46[a]	1.97[a]	1.64[a]	1.40[a]	1.22[a]
6	.168	.074	.040	.024	.016	.011	.008	.006	.004	3.94[a]	3.28[a]	2.79[a]
7	.236	.116	.068	.043	.029	.021	.016	.012	9.49[a]	7.67[a]	6.35[a]	5.35[a]
8	.296	.160	.099	.066	.046	.034	.026	.020	.016	.013	.011	9.00[a]
9	.349	.203	.131	.091	.066	.049	.038	.030	.024	.020	.016	.014
10	.396	.243	.164	.117	.086	.066	.052	.041	.034	.028	.023	.020
11	.437	.281	.196	.143	.108	.084	.067	.054	.044	.037	.031	.026
12	.473	.316	.226	.169	.130	.103	.083	.067	.056	.047	.040	.034
13	.505	.348	.255	.194	.152	.122	.099	.082	.068	.058	.049	.042
14	.534	.378	.283	.219	.174	.141	.116	.096	.081	.069	.059	.051
15	.560	.405	.309	.243	.195	.160	.133	.111	.095	.081	.070	.061
16	.583	.431	.334	.266	.216	.179	.149	.127	.108	.093	.081	.071
17	.603	.454	.357	.288	.236	.197	.166	.142	.122	.106	.092	.081
18	.622	.476	.379	.309	.256	.215	.183	.157	.136	.118	.104	.092
19	.639	.496	.399	.329	.275	.233	.199	.172	.149	.131	.115	.102
20	.655	.515	.419	.348	.293	.250	.215	.187	.163	.144	.127	.113
21	.669	.532	.437	.366	.310	.266	.230	.201	.177	.156	.139	.124
22	.683	.548	.454	.383	.327	.282	.246	.215	.190	.169	.150	.135
23	.695	.564	.470	.399	.343	.298	.260	.229	.203	.181	.162	.146
24	.706	.578	.486	.415	.359	.313	.275	.243	.216	.193	.173	.156
25	.717	.591	.500	.430	.374	.327	.289	.256	.229	.205	.185	.167
26	.727	.604	.514	.444	.388	.341	.302	.269	.241	.217	.196	.178
27	.736	.616	.527	.458	.401	.355	.315	.282	.253	.229	.207	.188
28	.744	.627	.540	.471	.415	.368	.328	.294	.265	.240	.218	.199
29	.752	.638	.552	.483	.427	.380	.340	.306	.277	.251	.229	.209
30	.760	.648	.563	.495	.439	.392	.352	.318	.288	.262	.239	.219
40	.816	.724	.651	.591	.539	.494	.454	.419	.387	.359	.334	.311
60	.875	.808	.752	.704	.661	.623	.587	.555	.526	.498	.473	.449
80	.905	.853	.808	.769	.733	.700	.670	.641	.615	.590	.566	.544
100	.924	.881	.844	.810	.780	.751	.725	.700	.676	.654	.632	.612
120	.936	.900	.868	.839	.813	.788	.764	.742	.721	.700	.681	.663
140	.945	.913	.886	.861	.837	.815	.794	.774	.755	.736	.719	.702
170	.955	.928	.905	.884	.864	.845	.827	.809	.792	.776	.761	.746
200	.961	.939	.919	.900	.883	.866	.850	.835	.820	.806	.792	.779
240	.968	.949	.932	.916	.901	.887	.873	.860	.848	.835	.823	.811
320	.976	.961	.948	.936	.925	.914	.903	.893	.883	.873	.864	.854
440	.982	.972	.962	.953	.945	.937	.929	.921	.913	.906	.899	.891
600	.987	.979	.972	.966	.959	.953	.947	.941	.936	.930	.924	.919
800	.990	.984	.979	.974	.969	.965	.960	.956	.951	.947	.943	.939
1000	.992	.987	.983	.979	.975	.972	.968	.964	.961	.957	.954	.950

[a] Multiply entry by 10^{-3}.

(continued)

Table A.9. (*Continued*)

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p = 4$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.001[a]	.001[a]	.001[a]	.002[a]	.002[a]	.002[a]	.003[a]
4	1.38[a]	.292[a]	.127[a]	.075[a]	.052[a]	.040[a]	.033[a]	.029[a]	.026[a]	.025[a]	.023[a]	.022[a]
5	.026	6.09[a]	2.31[a]	1.13[a]	.647[a]	.416[a]	.292[a]	.218[a]	.172[a]	.141[a]	.120[a]	.105[a]
6	.076	.024	.010	5.07[a]	2.90[a]	1.82[a]	1.22[a]	.872[a]	.652[a]	.508[a]	.409[a]	.338[a]
7	.135	.051	.024	.013	7.74[a]	4.94[a]	3.34[a]	2.36[a]	1.74[a]	1.33[a]	1.05[a]	.848[a]
8	.194	.084	.043	.025	.015	.010	6.98[a]	4.99[a]	3.70[a]	2.82[a]	2.21[a]	1.77[a]
9	.249	.119	.066	.040	.026	.017	.012	8.91[a]	6.66[a]	5.11[a]	4.01[a]	3.21[a]
10	.298	.155	.091	.057	.038	.027	.019	.014	.011	8.29[a]	6.54[a]	5.25[a]
11	.343	.190	.117	.077	.053	.037	.027	.021	.016	.012	9.84[a]	7.95[a]
12	.382	.223	.143	.097	.068	.049	.037	.028	.022	.017	.014	.011
13	.418	.255	.169	.117	.085	.063	.047	.037	.029	.023	.019	.015
14	.450	.286	.194	.138	.102	.077	.059	.046	.037	.030	.024	.020
15	.479	.314	.219	.159	.119	.091	.071	.056	.045	.037	.030	.025
16	.506	.340	.243	.180	.136	.106	.083	.067	.054	.044	.037	.031
17	.529	.365	.266	.200	.154	.121	.096	.078	.064	.053	.044	.037
18	.551	.389	.288	.219	.171	.136	.109	.089	.074	.061	.051	.044
19	.571	.410	.309	.239	.188	.151	.123	.101	.084	.070	.059	.051
20	.589	.431	.329	.257	.205	.166	.136	.113	.094	.079	.068	.058
21	.606	.450	.348	.275	.221	.181	.149	.124	.105	.089	.076	.065
22	.621	.468	.366	.292	.237	.195	.162	.136	.115	.098	.085	.073
23	.636	.485	.383	.309	.253	.210	.175	.148	.126	.108	.093	.081
24	.649	.501	.399	.325	.268	.224	.188	.160	.137	.118	.102	.089
25	.661	.516	.415	.340	.283	.237	.201	.172	.148	.128	.111	.097
26	.673	.530	.430	.355	.297	.251	.214	.183	.158	.138	.120	.106
27	.684	.544	.444	.369	.311	.264	.226	.195	.169	.147	.129	.114
28	.694	.556	.458	.383	.324	.277	.238	.206	.180	.157	.138	.122
29	.703	.568	.471	.396	.337	.289	.250	.217	.190	.167	.147	.131
30	.712	.580	.483	.409	.349	.301	.261	.228	.200	.177	.157	.139
40	.779	.668	.583	.513	.455	.406	.364	.327	.295	.267	.243	.221
60	.849	.767	.700	.643	.592	.547	.507	.471	.438	.409	.382	.357
80	.885	.821	.766	.718	.675	.636	.600	.567	.536	.508	.482	.457
100	.908	.854	.809	.768	.730	.696	.664	.634	.606	.580	.555	.532
120	.923	.877	.838	.802	.770	.739	.711	.684	.658	.634	.611	.590
140	.934	.894	.860	.828	.799	.772	.746	.721	.698	.676	.655	.635
170	.945	.912	.883	.856	.831	.808	.785	.764	.743	.724	.705	.687
200	.953	.925	.900	.876	.855	.834	.814	.795	.777	.759	.742	.726
240	.961	.937	.916	.896	.877	.859	.842	.826	.810	.795	.780	.765
320	.971	.952	.936	.921	.907	.893	.879	.866	.854	.841	.829	.818
440	.979	.965	.953	.942	.931	.921	.911	.901	.891	.882	.872	.863
600	.984	.974	.966	.957	.949	.941	.934	.926	.919	.912	.905	.898
800	.988	.981	.974	.968	.961	.956	.950	.944	.938	.933	.927	.922
1000	.991	.985	.979	.974	.969	.964	.960	.955	.950	.946	.941	.937

[a] Multiply entry by 10^{-3}.

(continued)

Table A.9. (*Continued*)

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p = 5$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	$.001^a$	$.001^a$	$.001^a$	$.001^a$	$.001^a$	$.001^a$	$.001^a$	$.001^a$
5	1.60^a	$.291^a$	$.105^a$	$.052^a$	$.031^a$	$.021^a$	$.015^a$	$.012^a$	$.010^a$	$.008^a$	$.007^a$	$.007^a$
6	.021	4.39^a	1.48^a	$.647^a$	$.335^a$	$.197^a$	$.126^a$	$.087^a$	$.064^a$	$.049^a$	$.039^a$	$.032^a$
7	.063	.017	6.36^a	2.90^a	1.51^a	$.872^a$	$.544^a$	$.361^a$	$.253^a$	$.185^a$	$.141^a$	$.110^a$
8	.114	.037	.016	7.74^a	4.21^a	2.48^a	1.56^a	1.03^a	$.716^a$	$.516^a$	$.385^a$	$.296^a$
9	.165	.063	.029	.015	8.79^a	5.35^a	3.43^a	2.30^a	1.61^a	1.16^a	$.861^a$	$.657^a$
10	.215	.092	.046	.026	.015	9.64^a	6.34^a	4.34^a	3.06^a	2.22^a	1.66^a	1.27^a
11	.261	.122	.066	.038	.024	.015	.010	7.22^a	5.17^a	3.80^a	2.86^a	2.19^a
12	.303	.153	.086	.053	.034	.022	.015	.011	7.99^a	5.95^a	4.51^a	3.49^a
13	.341	.183	.108	.068	.045	.031	.022	.016	.012	8.68^a	6.66^a	5.19^a
14	.376	.212	.130	.085	.057	.040	.029	.021	.016	.012	9.31^a	7.32^a
15	.407	.239	.152	.102	.070	.050	.037	.027	.021	.016	.012	9.88^a
16	.436	.266	.174	.119	.084	.061	.045	.034	.026	.020	.016	.013
17	.462	.291	.195	.136	.098	.072	.054	.042	.032	.025	.020	.016
18	.486	.315	.216	.154	.113	.084	.064	.050	.039	.031	.025	.020
19	.508	.337	.236	.171	.127	.096	.074	.058	.046	.037	.030	.024
20	.529	.359	.256	.188	.142	.109	.085	.067	.053	.043	.035	.029
21	.548	.379	.275	.205	.156	.121	.095	.076	.061	.050	.041	.034
22	.565	.398	.293	.221	.171	.134	.106	.085	.069	.057	.047	.039
23	.581	.416	.310	.237	.185	.146	.117	.095	.077	.064	.053	.044
24	.596	.433	.327	.253	.199	.159	.128	.104	.086	.071	.060	.050
25	.610	.449	.343	.268	.213	.171	.139	.114	.094	.079	.066	.056
26	.623	.465	.359	.283	.226	.183	.150	.124	.103	.087	.073	.062
27	.635	.479	.374	.297	.239	.195	.161	.134	.112	.094	.080	.068
28	.647	.493	.388	.311	.252	.207	.172	.143	.121	.102	.087	.075
29	.658	.506	.401	.324	.265	.219	.182	.153	.130	.110	.094	.081
30	.668	.519	.415	.337	.277	.230	.193	.163	.138	.118	.102	.088
40	.744	.617	.522	.446	.384	.333	.291	.255	.224	.198	.176	.156
60	.825	.729	.652	.587	.531	.482	.438	.400	.366	.336	.308	.284
80	.867	.791	.727	.672	.623	.578	.538	.502	.469	.438	.410	.385
100	.893	.830	.776	.728	.685	.645	.609	.576	.544	.516	.489	.464
120	.910	.856	.810	.768	.730	.694	.661	.631	.602	.575	.549	.525
140	.923	.876	.835	.798	.763	.731	.701	.673	.647	.621	.598	.575
170	.936	.897	.862	.830	.801	.773	.747	.722	.698	.675	.654	.633
200	.945	.912	.882	.854	.828	.803	.780	.758	.736	.716	.696	.677
240	.954	.926	.900	.877	.855	.833	.813	.793	.775	.757	.739	.722
300	.966	.944	.925	.906	.889	.872	.856	.841	.825	.811	.797	.783
440	.975	.959	.945	.931	.918	.905	.893	.881	.870	.858	.847	.836
600	.982	.970	.959	.949	.939	.930	.920	.911	.903	.894	.885	.877
800	.986	.977	.969	.961	.954	.947	.940	.933	.926	.919	.913	.906
1000	.989	.982	.975	.969	.963	.957	.951	.946	.940	.935	.929	.924

^{a} Multiply entry by 10^{-3}.

(continued)

Table A.9. (*Continued*)

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p=6$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.007[a]	.002[a]	.001[a]	.001[a]	.001[a]	.000	.000	.000	.000	.000	.000	.000
6	2.04[a]	.315[a]	.095[a]	.040[a]	.021[a]	.012[a]	.008[a]	.006[a]	.004[a]	.003[a]	.003[a]	.002[a]
7	.019	3.48[a]	1.05[a]	.416[a]	.197[a]	.106[a]	.063[a]	.040[a]	.027[a]	.020[a]	.015[a]	.011[a]
8	.054	.013	4.37[a]	1.82[a]	.872[a]	.465[a]	.270[a]	.168[a]	.111[a]	.076[a]	.055[a]	.041[a]
9	.098	.029	.011	4.94[a]	2.48[a]	1.36[a]	.798[a]	.497[a]	.325[a]	.222[a]	.157[a]	.115[a]
10	.144	.050	.021	.010	5.35[a]	3.04[a]	1.83[a]	1.16[a]	.762[a]	.521[a]	.369[a]	.269[a]
11	.189	.074	.034	.017	9.64[a]	5.67[a]	3.51[a]	2.26[a]	1.51[a]	1.05[a]	.744[a]	.543[a]
12	.232	.099	.049	.027	.015	9.35[a]	5.94[a]	3.92[a]	2.66[a]	1.86[a]	1.34[a]	.983[a]
13	.271	.126	.066	.037	.022	.014	9.17[a]	6.17[a]	4.27[a]	3.03[a]	2.20[a]	1.63[a]
14	.308	.152	.084	.049	.031	.020	.013	9.07[a]	6.38[a]	4.59[a]	3.37[a]	2.52[a]
15	.341	.179	.103	.063	.040	.026	.018	.013	9.00[a]	6.57[a]	4.88[a]	3.68[a]
16	.372	.204	.122	.077	.050	.034	.024	.017	.012	8.97[a]	6.74[a]	5.14[a]
17	.400	.229	.141	.091	.061	.042	.030	.021	.016	.012	8.97[a]	6.90[a]
18	.426	.252	.160	.106	.072	.051	.037	.027	.020	.015	.012	8.97[a]
19	.450	.275	.179	.121	.084	.060	.044	.033	.025	.019	.015	.011
20	.473	.296	.197	.136	.096	.070	.052	.039	.030	.023	.018	.014
21	.493	.317	.215	.151	.109	.080	.060	.045	.035	.027	.021	.017
22	.512	.337	.233	.166	.121	.090	.068	.052	.041	.032	.025	.020
23	.530	.355	.250	.181	.134	.101	.077	.060	.047	.037	.030	.024
24	.546	.373	.266	.195	.146	.111	.086	.067	.053	.042	.034	.028
25	.562	.390	.282	.210	.159	.122	.095	.075	.060	.048	.039	.032
26	.576	.406	.298	.224	.171	.133	.104	.083	.066	.054	.044	.036
27	.590	.422	.313	.237	.183	.143	.113	.091	.073	.060	.049	.040
28	.603	.436	.327	.251	.195	.154	.123	.099	.080	.066	.054	.045
29	.615	.450	.341	.264	.207	.165	.132	.107	.088	.072	.060	.050
30	.626	.464	.355	.277	.219	.175	.142	.116	.095	.079	.066	.055
40	.711	.570	.467	.387	.324	.273	.232	.198	.170	.147	.127	.110
60	.802	.693	.608	.536	.476	.424	.379	.340	.305	.275	.249	.225
80	.849	.762	.690	.629	.574	.526	.483	.445	.410	.378	.350	.324
100	.878	.806	.745	.691	.642	.599	.559	.523	.489	.458	.430	.404
120	.898	.836	.783	.735	.692	.652	.616	.582	.551	.521	.494	.468
140	.912	.858	.811	.769	.730	.694	.660	.629	.599	.572	.546	.521
170	.927	.882	.842	.806	.772	.740	.710	.682	.656	.630	.607	.584
200	.938	.899	.864	.832	.803	.774	.748	.722	.698	.675	.653	.632
240	.948	.915	.886	.858	.833	.808	.785	.763	.741	.721	.701	.682
320	.961	.936	.913	.892	.872	.852	.834	.816	.799	.782	.766	.750
440	.972	.953	.936	.920	.905	.890	.876	.862	.849	.836	.823	.811
600	.979	.965	.953	.941	.930	.918	.908	.897	.887	.877	.867	.857
800	.984	.974	.964	.955	.947	.938	.930	.922	.914	.906	.898	.891
1000	.987	.979	.971	.964	.957	.950	.944	.937	.930	.924	.918	.912

[a] Multiply entry by 10^{-3}.

(continued)

Table A.9. (*Continued*)

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p = 7$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
6	.043[a]	.006[a]	.002[a]	.001[a]	.001[a]	.000	.000	.000	.000	.000	.000	.000
7	2.62[a]	.350[a]	.091[a]	.033[a]	.015[a]	.008[a]	.005[a]	.003[a]	.002[a]	.002[a]	.001[a]	.001[a]
8	.018	2.95[a]	.809[a]	.292[a]	.126[a]	.063[a]	.034[a]	.020[a]	.013[a]	.009[a]	.006[a]	.005[a]
9	.048	.010	3.20[a]	1.22[a]	.543[a]	.270[a]	.147[a]	.086[a]	.053[a]	.035[a]	.024[a]	.017[a]
10	.087	.023	8.07[a]	3.34[a]	1.56[a]	.798[a]	.440[a]	.259[a]	.160[a]	.104[a]	.070[a]	.049[a]
11	.128	.040	.016	6.97[a]	3.43[a]	1.83[a]	1.04[a]	.619[a]	.387[a]	.252[a]	.170[a]	.119[a]
12	.170	.060	.026	.012	6.34[a]	3.51[a]	2.05[a]	1.25[a]	.796[a]	.525[a]	.357[a]	.249[a]
13	.209	.083	.038	.019	.010	5.94[a]	3.57[a]	2.23[a]	1.45[a]	.967[a]	.665[a]	.468[a]
14	.246	.106	.052	.027	.015	9.17[a]	5.67[a]	3.63[a]	2.40[a]	1.62[a]	1.13[a]	.804[a]
15	.281	.129	.067	.037	.022	.013	8.37[a]	5.48[a]	3.68[a]	2.54[a]	1.79[a]	1.28[a]
16	.313	.153	.083	.047	.029	.018	.012	7.80[a]	5.34[a]	3.73[a]	2.66[a]	1.94[a]
17	.343	.176	.099	.059	.037	.024	.016	.011	7.38[a]	5.24[a]	3.78[a]	2.78[a]
18	.370	.199	.116	.071	.045	.030	.020	.014	9.81[a]	7.06[a]	5.16[a]	3.83[a]
19	.396	.221	.133	.083	.054	.037	.025	.018	.013	9.20[a]	6.80[a]	5.10[a]
20	.420	.242	.149	.096	.064	.044	.031	.022	.016	.012	8.72[a]	6.60[a]
21	.442	.263	.166	.109	.074	.052	.037	.026	.019	.014	.011	8.34[a]
22	.462	.283	.183	.123	.085	.060	.043	.031	.023	.018	.013	.010
23	.482	.301	.199	.136	.095	.068	.050	.037	.028	.021	.016	.013
24	.499	.320	.215	.149	.106	.077	.057	.042	.032	.025	.019	.015
25	.516	.337	.230	.162	.117	.086	.064	.048	.037	.029	.022	.018
26	.532	.354	.246	.175	.128	.095	.071	.055	.042	.033	.026	.020
27	.547	.370	.260	.188	.139	.104	.079	.061	.047	.037	.029	.024
28	.561	.385	.275	.201	.150	.113	.087	.068	.053	.042	.033	.027
29	.574	.399	.289	.214	.161	.123	.095	.074	.059	.047	.037	.030
30	.586	.413	.302	.226	.172	.132	.103	.081	.064	.052	.042	.034
40	.679	.526	.417	.335	.273	.224	.185	.154	.128	.108	.091	.077
60	.779	.660	.566	.490	.426	.373	.327	.288	.254	.225	.200	.178
80	.832	.735	.656	.588	.530	.479	.434	.394	.358	.326	.298	.272
100	.864	.783	.715	.656	.603	.556	.513	.475	.439	.408	.378	.352
120	.886	.817	.757	.704	.657	.613	.574	.537	.504	.473	.444	.418
140	.902	.841	.788	.741	.698	.658	.621	.587	.556	.526	.498	.472
170	.919	.868	.823	.782	.744	.709	.676	.645	.616	.589	.563	.539
200	.931	.887	.848	.812	.778	.747	.717	.689	.662	.637	.613	.590
240	.942	.905	.871	.841	.812	.784	.758	.733	.709	.687	.665	.644
320	.957	.928	.902	.878	.855	.833	.812	.792	.773	.754	.736	.719
440	.968	.947	.928	.910	.893	.876	.860	.844	.829	.814	.800	.786
600	.977	.961	.947	.933	.920	.908	.895	.883	.872	.860	.849	.838
800	.982	.971	.960	.950	.940	.930	.920	.911	.902	.893	.884	.876
1000	.986	.977	.968	.959	.951	.943	.936	.928	.921	.914	.906	.899

[a] Multiply entry by 10^{-3}.

(continued)

Table A.9. ***(Continued)***

ν_E	ν_H 1	2	3	4	5	6	7	8	9	10	11	12
						$p = 8$						
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
6	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
7	.138[a]	.015[a]	.004[a]	.001[a]	.001[a]	.000	.000	.000	.000	.000	.000	.000
8	3.30[a]	.393[a]	.090[a]	.029[a]	.012[a]	.006[a]	.003[a]	.002[a]	.001[a]	.001[a]	.001[a]	.000
9	.017	2.63[a]	.659[a]	.218[a]	.087[a]	.040[a]	.020[a]	.011[a]	.007[a]	.004[a]	.003[a]	.002[a]
10	.044	8.63[a]	2.46[a]	.872[a]	.361[a]	.168[a]	.086[a]	.047[a]	.028[a]	.017[a]	.011[a]	.008[a]
11	.078	.019	6.15[a]	2.36[a]	1.03[a]	.497[a]	.259[a]	.144[a]	.085[a]	.052[a]	.034[a]	.023[a]
12	.116	.033	.012	4.99[a]	2.30[a]	1.16[a]	.619[a]	.351[a]	.209[a]	.130[a]	.084[a]	.056[a]
13	.154	.051	.020	8.91[a]	4.34[a]	2.26[a]	1.25[a]	.727[a]	.441[a]	.278[a]	.181[a]	.122[a]
14	.190	.070	.030	.014	7.22[a]	3.92[a]	2.23[a]	1.33[a]	.824[a]	.527[a]	.347[a]	.235[a]
15	.225	.090	.041	.021	.011	6.17[a]	3.63[a]	2.22[a]	1.40[a]	.910[a]	.608[a]	.416[a]
16	.258	.111	.054	.028	.016	9.06[a]	5.48[a]	3.42[a]	2.20[a]	1.46[a]	.987[a]	.683[a]
17	.289	.133	.067	.037	.021	.013	7.80[a]	4.98[a]	3.27[a]	2.20[a]	1.51[a]	1.06[a]
18	.318	.154	.082	.046	.027	.017	.011	6.92[a]	4.62[a]	3.15[a]	2.19[a]	1.56[a]
19	.345	.175	.096	.056	.034	.021	.014	9.23[a]	6.26[a]	4.34[a]	3.06[a]	2.19[a]
20	.370	.195	.111	.067	.042	.027	.018	.012	8.22[a]	5.77[a]	4.12[a]	2.99[a]
21	.393	.215	.127	.078	.050	.033	.022	.015	.010	7.46[a]	5.39[a]	3.95[a]
22	.415	.235	.142	.089	.058	.039	.026	.018	.013	9.40[a]	6.86[a]	5.08[a]
23	.436	.254	.157	.101	.067	.045	.031	.022	.016	.012	8.56[a]	6.39[a]
24	.455	.272	.172	.113	.076	.052	.037	.026	.019	.014	.010	7.88[a]
25	.473	.289	.187	.124	.085	.060	.042	.031	.023	.017	.013	9.56[a]
26	.490	.306	.201	.136	.095	.067	.048	.035	.026	.020	.015	.011
27	.505	.322	.215	.148	.104	.075	.055	.040	.030	.023	.017	.013
28	.520	.338	.229	.160	.114	.083	.061	.045	.034	.026	.020	.016
29	.534	.353	.243	.172	.124	.091	.068	.051	.039	.030	.023	.018
30	.548	.367	.256	.183	.134	.099	.074	.056	.043	.034	.026	.021
40	.649	.485	.372	.290	.229	.182	.146	.118	.096	.079	.065	.054
60	.758	.627	.527	.447	.381	.327	.282	.244	.212	.184	.161	.141
80	.815	.709	.623	.551	.489	.435	.389	.348	.313	.281	.253	.229
100	.851	.761	.687	.622	.566	.516	.471	.431	.395	.362	.333	.306
120	.875	.798	.732	.675	.623	.577	.535	.496	.461	.429	.399	.372
140	.892	.825	.767	.715	.667	.625	.585	.549	.515	.484	.455	.428
170	.911	.854	.804	.759	.717	.679	.644	.610	.579	.550	.523	.497
200	.924	.875	.831	.791	.755	.720	.688	.657	.629	.602	.576	.551
240	.936	.895	.858	.823	.791	.761	.732	.705	.679	.655	.631	.609
320	.952	.920	.891	.865	.839	.815	.792	.770	.748	.728	.708	.689
440	.965	.942	.920	.900	.880	.862	.844	.827	.810	.794	.778	.762
600	.974	.957	.941	.926	.911	.897	.883	.870	.857	.844	.831	.819
800	.981	.968	.955	.944	.933	.922	.911	.901	.890	.880	.871	.861
1000	.985	.974	.964	.955	.946	.937	.928	.920	.911	.903	.895	.887

[a] Multiply entry by 10^{-3}.