STA218
Introduction to Estimation

Al Nosedal.
University of Toronto.
Fall 2018

October 24, 2018

Confidence Interval Estimator of μ

$$\left(\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

The probability $1 - \alpha$ is called the **confidence level**.
$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is called the **lower confidence limit (LCL)**.
$\bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ is called the **upper confidence limit (UCL)**.

Confidence Interval Estimator of μ.

We often represent the confidence interval estimator as
$\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
where the minus sign defines the lower confidence limit and the plus sign defines the upper confidence limit.

Example

In an effort to estimate the mean amount spent per customer for dinner at a major Atlanta restaurant, data were collected for a sample of 49 customers. Assume a population standard deviation of \$5.

a. At 95% confidence, what is the margin of error?

b. If the sample mean is \$ 24.80, what is the 95% confidence interval for the population mean?

Solution

In this case, μ = mean amount spent per customer for dinner for **all** customers at a major Atlanta restaurant.

a. margin of error $= Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (1.96)\left(\frac{5}{7}\right) = 1.4$

b. $\left(\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$

$(24.80 - 1.4, 24.80 + 1.4)$

$(23.40, 26.20)$ 95% Confidence Interval for μ.

Example

Playbill magazine reported that the mean annual household income of its readers is \$ 119,155. Assume this estimate of the mean annual household income is based on a sample of 80 households, and based on past studies, the population standard deviation is known to be $\sigma = \$\ 30{,}000$.

a. Develop a 90 % confidence interval estimate of the population mean.

b. Develop a 95 % confidence interval estimate of the population mean.

c. Develop a 99 % confidence interval estimate of the population mean.

Solution

In this case, $\mu =$ mean annual household income of **all** its readers. $\sigma = 30,000$, $n = 80$, $1 - \alpha = 0.90$, $\alpha = 0.10$.
a) margin of error =
$Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (1.64)\left(\frac{30000}{\sqrt{80}}\right) = 5500.727 \approx 5500.73$
Confidence Interval is given by: estimate $\pm$ margin of error. That is: $\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.
$119,155 \pm 5500.73$

$(113,654.27,\ 124,655.73)$

Solution

In this case, μ = mean annual household income of **all** its readers.
$\sigma = 30,000$, $n = 80$, $1 - \alpha = 0.95$, $\alpha = 0.05$.
b) margin of error $= Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (1.96)\left(\frac{30000}{\sqrt{80}}\right) = 6574.04$
Confidence Interval is given by: estimate $\pm$ margin of error. That is: $\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.
$119,155 \pm 6574.04$

$(112,580.96,\ 125,729.04)$

Solution

In this case, μ = mean annual household income of **all** its readers.
$\sigma = 30,000$, $n = 80$, $1 - \alpha = 0.99$, $\alpha = 0.01$.
c) margin of error =
$Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = (2.57)\left(\frac{30000}{\sqrt{80}}\right) = 8620.042 \approx 8620.04$
Confidence Interval is given by: estimate $\pm$ margin of error. That is: $\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.
$119,155 \pm 8620.04$

$(110,534.96,\ 127,775.04)$

Example

The number of cars sold annually by used car salespeople is Normally distributed with a standard deviation of 15. A random sample of 15 salespeople was taken, and the number of cars each sold is listed here. Find the 95% confidence interval estimate of the population mean. Interpret the interval estimate.

Example

79	43	58	66	101
63	79	33	58	71
60	101	74	55	88

R Code

```
simple.z.test = function(x,sigma,conf.level=0.95) {
n = length(x);
xbar=mean(x);
alpha = 1 - conf.level;
zstar = qnorm(1-alpha/2);
SE = sigma/sqrt(n);
xbar + c(-zstar*SE,zstar*SE);
}
```

R Code

```
# Step 1. Entering data;

cars=c(79, 43, 58, 66, 101, 63, 79,
33, 58, 71, 60, 101, 74, 55, 88);

# Step 2. Finding CI;

simple.z.test(cars,15);
```

R Code

```
## [1] 61.00909 76.19091
```

We estimate that the mean number of cars sold annually by all used car salespersons lies between 61 and 76, approximately. This type of estimate is correct 95% of the time.

Example

Scheer Industries is considering a new computer-assisted program to train maintenance employees to do machine repairs. In order to fully evaluate the program, the director of manufacturing requested an estimate of the population mean time required for maintenance employees to complete the computer-assisted training. Use 6.84 days as a planning value for the population standard deviation.
a. Assuming 95% confidence, what sample size would be required to obtain a margin of error of 1.5 days?
b. If the precision statement was made with 90% confidence, what sample size would be required to obtain a margin of error of 2 days?

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{B} \right)^2 .$$

B stands for the **bound on the error of estimation**.

Example

Smith Travel Research provides information on the one-night cost of hotel rooms through-out the United States. Use \$ 2 as the desired margin of error and \$ 22.50 as the planning value for the population standard deviation to find the sample size recommended in a), b), and c).
a. A 90% confidence interval estimate of the population mean cost of hotel rooms.
b. A 95% confidence interval estimate of the population mean cost of hotel rooms.
c. A 99% confidence interval estimate of the population mean cost of hotel rooms.

Solution

a) Confidence level $= 0.90$ $(1 - \alpha = 0.90)$, $B = 2$, $\sigma = 22.50$.
From Table 3, $Z_{0.05} = 1.65$

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{B} \right)^2 = \left(\frac{(1.65)(22.50)}{2} \right)^2 \approx 344.5664$$

Final answer: **345**.

Solution

b) Confidence level $= 0.95$ $(1-\alpha = 0.95)$, $B = 2$, $\sigma = 22.50$.
From Table 3, $Z_{0.025} = 1.96$

$$n = \left(\frac{Z_{\frac{\alpha}{2}}\sigma}{B}\right)^2 = \left(\frac{(1.96)(22.50)}{2}\right)^2 \approx 486.2025$$

Final answer: **487**.

Solution

c) Confidence level $= 0.99$ $(1 - \alpha = 0.99)$, $B = 2$, $\sigma = 22.50$.
From Table 3, $Z_{0.005} = 2.58$

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{B} \right)^2 = \left(\frac{(2.58)(22.50)}{2} \right)^2 \approx 842.4506$$

Final answer: **843**.

Exercise 10.57

A medical statistician wants to estimate the average weight loss of people who are on a new diet plan. In a preliminary study, he guesses that the standard deviation of the population of weight losses is about 10 pounds. How large a sample should he take to estimate the mean weight loss to within 2 pounds, with 90% confidence?

Solution

Confidence level $= 0.90$ $(1-\alpha = 0.90)$, $B = 2$, $\sigma = 10$. From Table 3, $Z_{0.05} = 1.65$

$$n = \left(\frac{Z_{\frac{\alpha}{2}}\sigma}{B}\right)^2 = \left(\frac{(1.65)(10)}{2}\right)^2 \approx 68.0625$$

Final answer: **69**.