

Probability Review

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This is an assignment that all of you should attempt by yourself after the first class in the course. All of you are expected to be very familiar with concepts from your Statistics pre-requisite course STA107H5/STA256H5. If you're not feeling comfortable with these questions, I would strongly encourage you to go through this chapter and exercises in their entirety:

<http://jeffe.cs.illinois.edu/teaching/algorithms/notes/01-random.pdf>

Problem 1 (Card Deck). *Two cards are picked uniformly randomly from a deck.*

1. *What is the probability that the first card is a Queen of either Spades(♠) or Clubs(♣)?*
2. *What is the probability that the first card is either a Queen, or its suit is Spades?*
3. *What is the probability that the first card is Spades? Conditioned on the first card being Spades, what is the probability that the second card is a Queen?*

Problem 2 (Pecking Chickens). *10 chickens are arranged in a circle. Each chicken independently and uniformly picks a direction from LEFT/RIGHT, and pecks the neighboring chicken in that direction.*

1. *What is the expected number of chickens that are not being pecked by either of their neighbors?*
2. *What if there were n chickens instead of 10?*

Problem 3 (Tossing Coins). *You are given a fair coin.*

1. *Suppose you start tossing the coin repeatedly, and stop when you see the first HEADS. Let X denote the number of tosses until you stop. What is the expectation of X , denoted $\mathbb{E}[X]$? What is its variance?*
2. *Suppose you start tossing the coin repeatedly, and stop when you see two consecutive HEADS for the first time. What is the expected number of tosses until you stop?*

Problem 4 (Collection of Dice [MU05]). *Suppose that we roll 10 standard six-sided dice. What is the probability that their sum is divisible by 6, assuming that the rolls are independent?*

References

- [MU05] Michael Mitzenmacher and Eli Upfal. *Probability and Computing: Randomized Algorithms and Probabilistic Analysis*. Cambridge University Press, 2005.