CSC263 Winter 2020

Dictionary ADT

Week 3

Today and next two weeks

ADT Dictionary

Data structure **Binary search tree (BST)**
Balanced BST - AVL tree
Hash Table

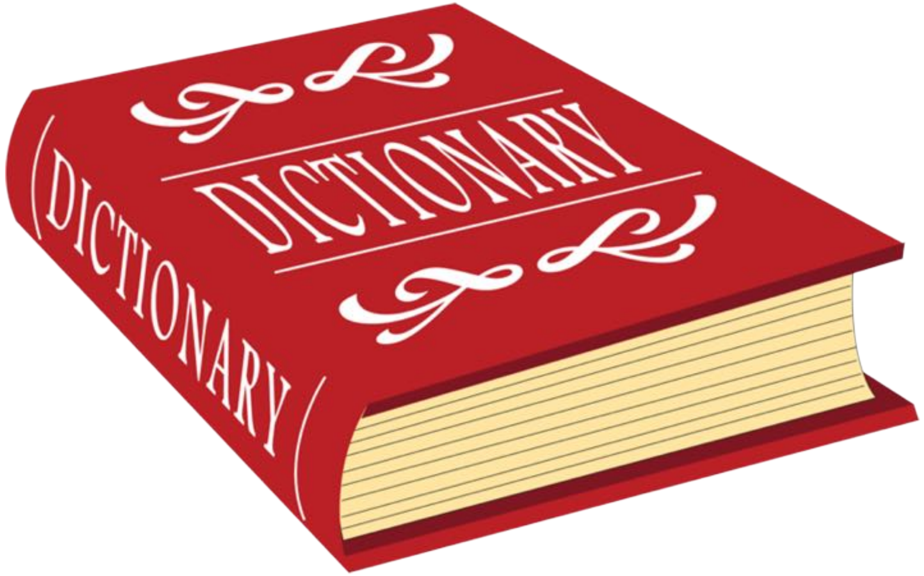
Dictionary

What's stored

- words

Supported operations

- Search for a word
- Insert a word
- Delete a word

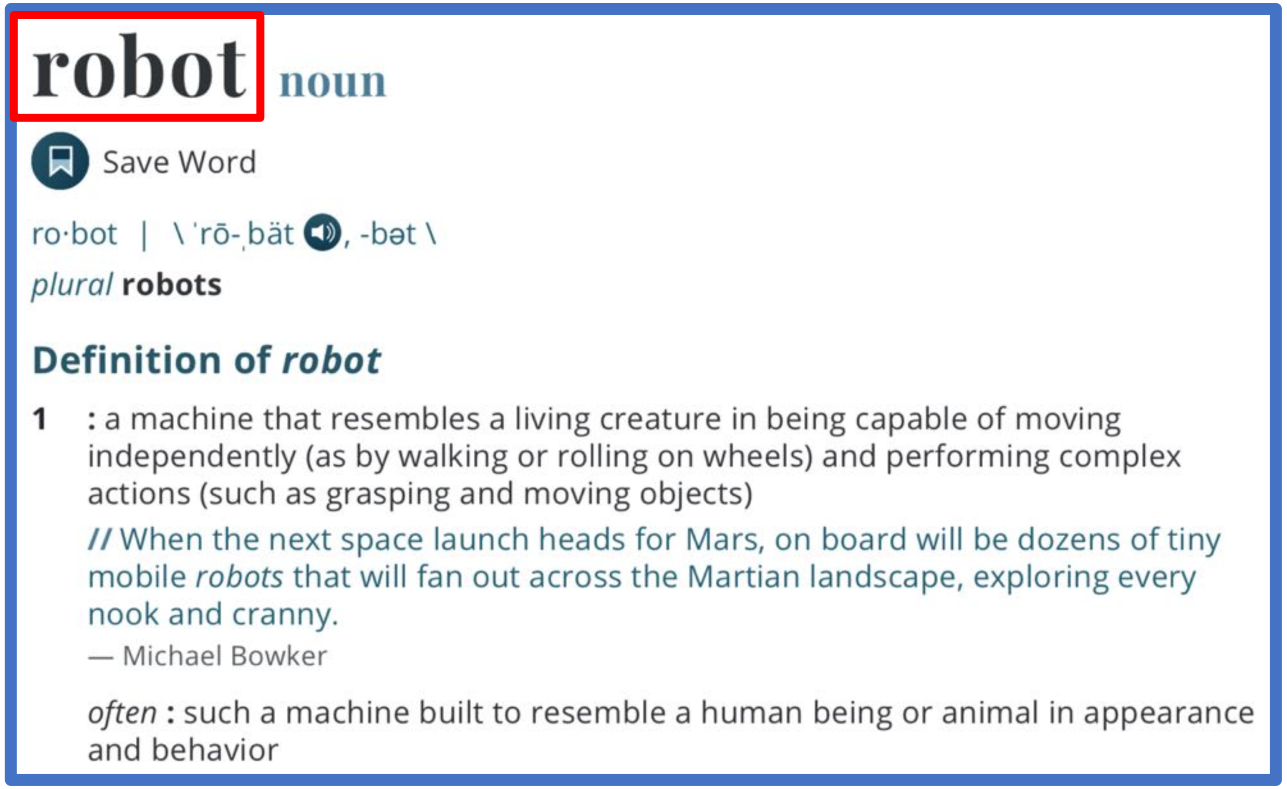


Dictionary, more precisely

What's stored

A set **S** of where each item/node **x** has a field **x.key**
(assumption: keys are distinct, unless otherwise specified)

Key k


robot noun

Save Word

ro·bot | \ ˈrō-ˌbät, -bət \

plural **robots**

Definition of *robot*

1 : a machine that resembles a living creature in being capable of moving independently (as by walking or rolling on wheels) and performing complex actions (such as grasping and moving objects)

// When the next space launch heads for Mars, on board will be dozens of tiny mobile *robots* that will fan out across the Martian landscape, exploring every nook and cranny.

— Michael Bowker

often : such a machine built to resemble a human being or animal in appearance and behavior

Node / item x

Dictionary ADT

Remember: k is a key, x is a node.

What's stored

A set **S** of where each item/node **x** has a field **x.key**

(assumption: keys are **distinct**, unless otherwise specified)

Supported operations

- **Search(S, k)** return **x** in **S**, s.t., **x.key = k**
  return NIL if no such **x**
- **Insert(S, x)** insert node **x** into **S**
  if already exists node **y** with same key, replace **y** with **x**
- **Delete(S, x)** delete a given node **x** from **S**

More on Delete

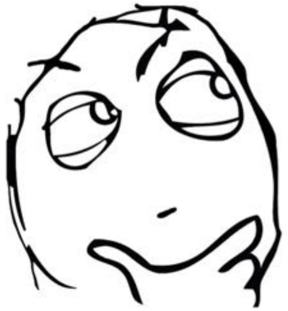
Why Delete(S, **x**) instead of Delete(S, k)?

Delete(S, **k**) can be implemented by:

```
x = Search(S, k)
Delete(S, x)
```

We want to separate different operations,
i.e., each operation focuses on only one job.

Think of use cases for an ADT Dictionary



1min > **Think** for yourself

1 min > **Pair** with your neighbor and talk about it

Share with the class

Implementing a Dictionary

using simple data structures

40 ⇄ 33 ⇄ 18 ⇄ 65 ⇄ 24 ⇄ 25

Unsorted (doubly) linked list

`Search(S,k)`

O(n) worst case

go through the list to find the key

`Insert(S,x)`

O(n) worst case

need to check if **x.key** is already in the list

`Delete(S,x)`

O(1) worst case

just delete, O(1) in a doubly linked list

Sorted array

[18 , 24 , 25 , 33 , 40 , 65]

`Search(S,k)`

O(log n) worst case

binary search!

`Insert(S,x)`

O(n) worst case

insert at front, everything has to shift to back

`Delete(S,x)`

O(n) worst case

delete, shift left

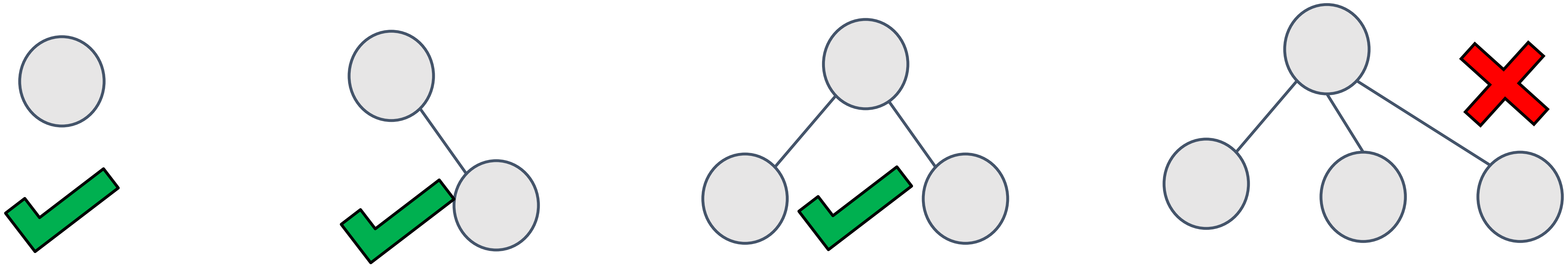
We can do better using smarter data structures!

	unsorted list	sorted array	BST	Balanced BST
`Search(S,k)`	O(n)	O(log n)	O(n)	O(log n)
`Insert(S,x)`	O(n)	O(n)	O(n)	O(log n)
`Delete(S,x)`	O(1)	O(n)	O(n)	O(log n)

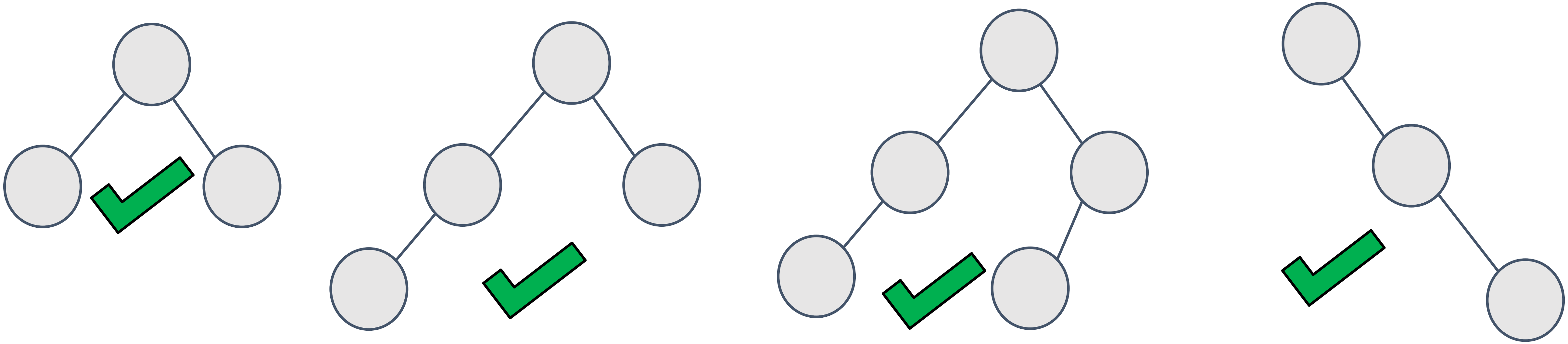
Binary Search Tree

It's a **binary** tree, like binary heap

Each node has **at most** 2 children



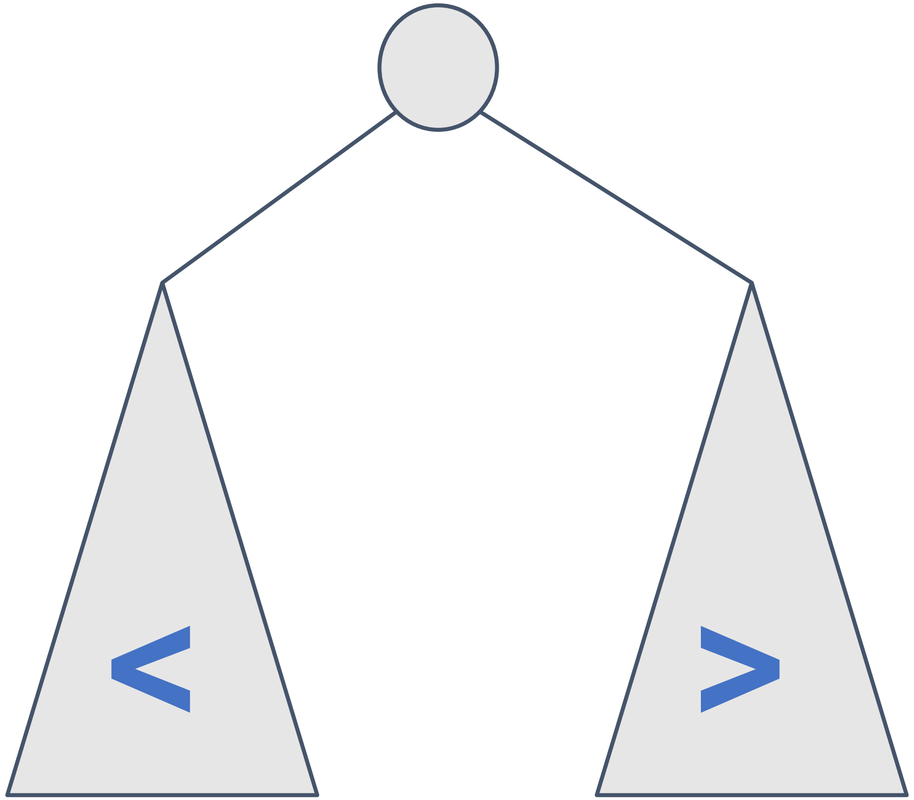
need NOT be nearly-complete, unlike binary heap



It has the BST property

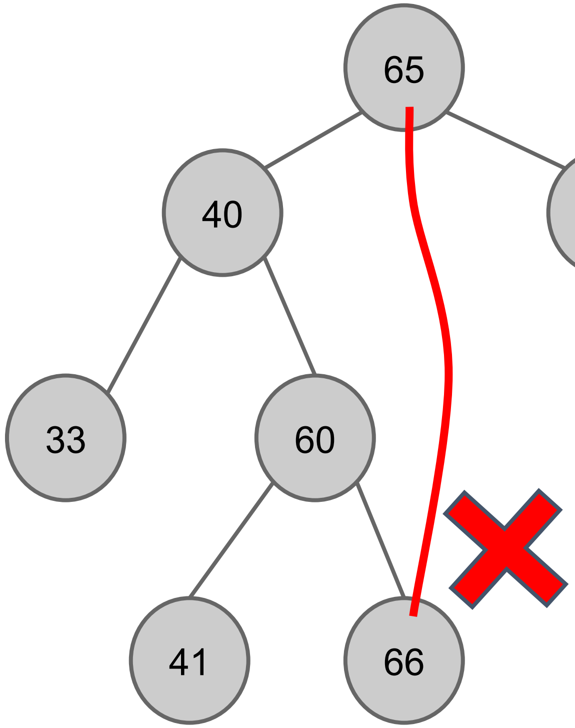
For **every** node **x** in the tree

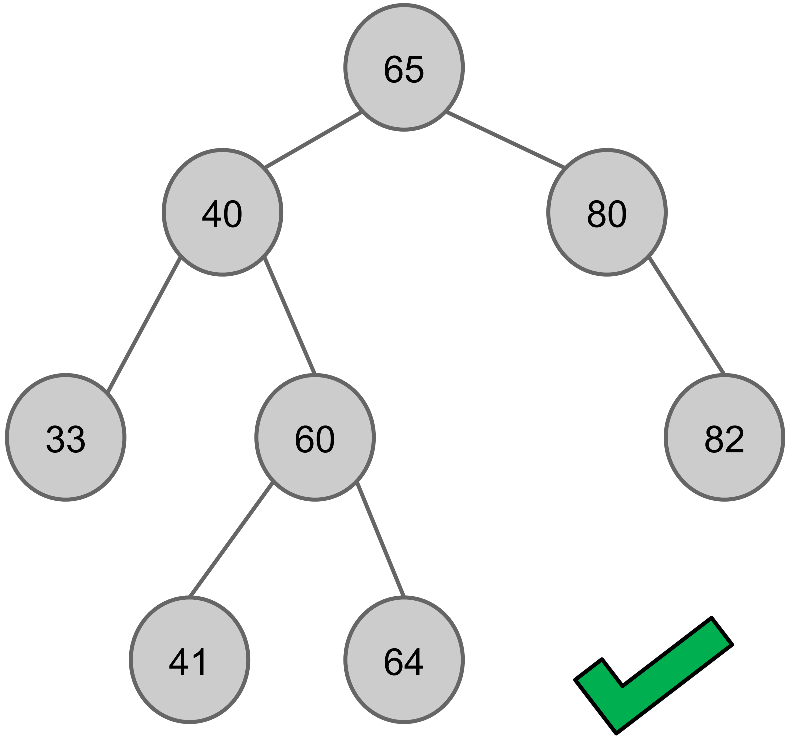
All nodes in the **left** subtree have keys **smaller** than **x.key**



All nodes in the **right** subtree have keys **larger** than **x.key**

BST or NOT?





Because of BST property,
we can say that the keys in a BST are sorted.

CSC148 Quiz: How to obtain a sorted list from a BST?

Perform an inorder traversal.

We pass a BST to a function by passing its **root** node.

```
InorderTraversal(x):
# print all keys in BST rooted at x in ascending order

    if x ≠ NIL:
        InorderTraversal(x.left)
        print x.key
        InorderTraversal(x.right)
```

Worst case running time of InorderTraversal:
O(n), because visit each node exactly once

Operations on a BST

Information at each node x

`x.key`	the key
`x.left`	the left child (node)
`x.right`	the right child (node)
`x.p`	the parent (node)
`x.value`	the value

Operations on a BST

read-only operations

`TreeSearch(root,k)`

`TreeMinimum(x) / TreeMaximum(x)`

`Successor(x) / Predecessor(x)`

modifying operations

`TreeInsert(root,x)`

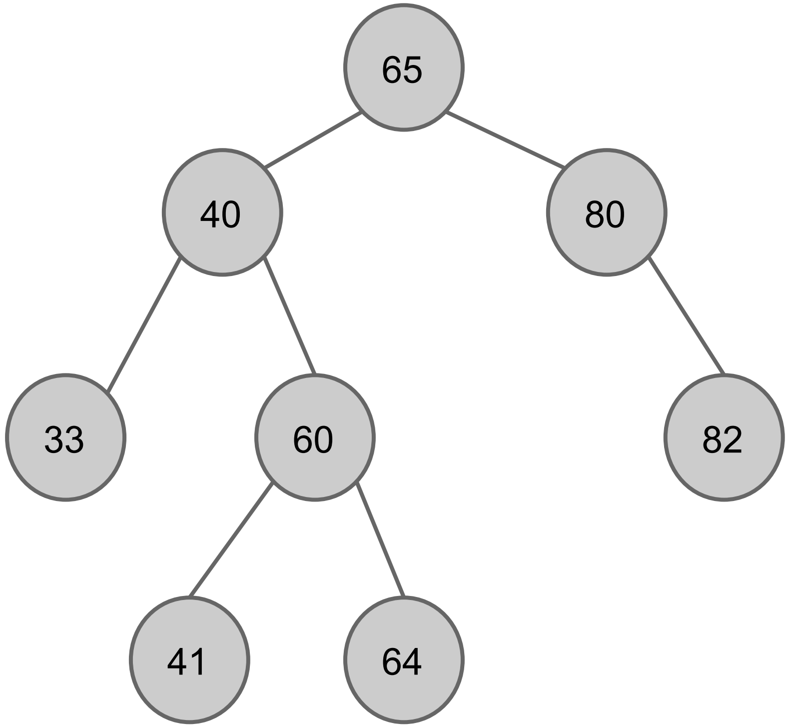
`TreeDelete(root,x)`

`TreeSearch(root,k)`

Search the BST rooted at root,
return the node with key k;
return NIL if not exist.

`TreeSearch(root,k)`

- start from root
- if **k** is **smaller** than the key of the current node, go **left**
- if **k** is **larger** than the key of the current node, go **right**
- if equal, **found**
- if going to **NIL**, **not found**



`TreeSearch(root,k)` Pseudo-code

```
TreeSearch(root, k):

   if root == NIL or k == root.key:
      return root
   if k < root.key:
      return TreeSearch(root.left, k)
   else:
      return TreeSearch(root.right, k)
```

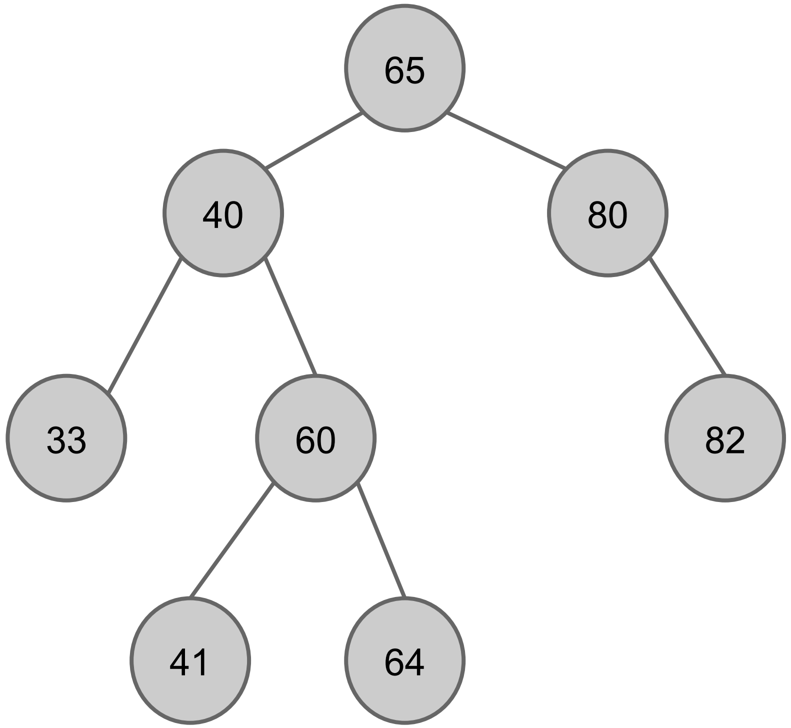
Worst case running time:
O(h), height of tree, going at most from root to leaf

TreeMinimum(x)

Return the node with the minimum key of the tree rooted at x

TreeMinimum(x)

- start from root
- keep going to the left, until cannot go anymore
- return that final node



`TreeMinimum(x)` Pseudo-code

```
TreeMinimum(x):

   while x.left ≠ NIL:
      x = x.left
   return x
```

Worst case running time:
O(h), height of tree, going at most from root to leaf

`TreeMaximum(x)` is exactly the same, except that it goes to the right instead of to the left.

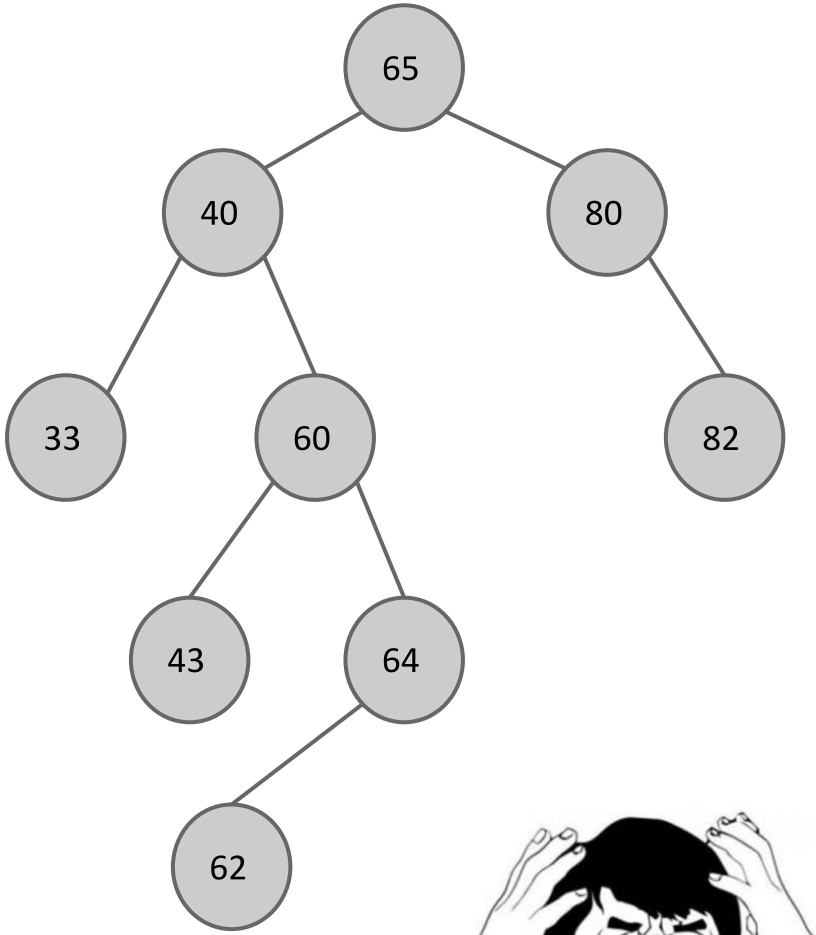
Successor(x)

Find the node which is the successor of x
in the sorted list obtained by inorder traversal

or, node with the smallest key larger than x

Successor(x)

- The successor of 33 is... 40
- The successor of 40 is… 43
- The successor of 64 is… 65
- The successor of 65 is … 80



`Successor(x)`
Organize into two cases

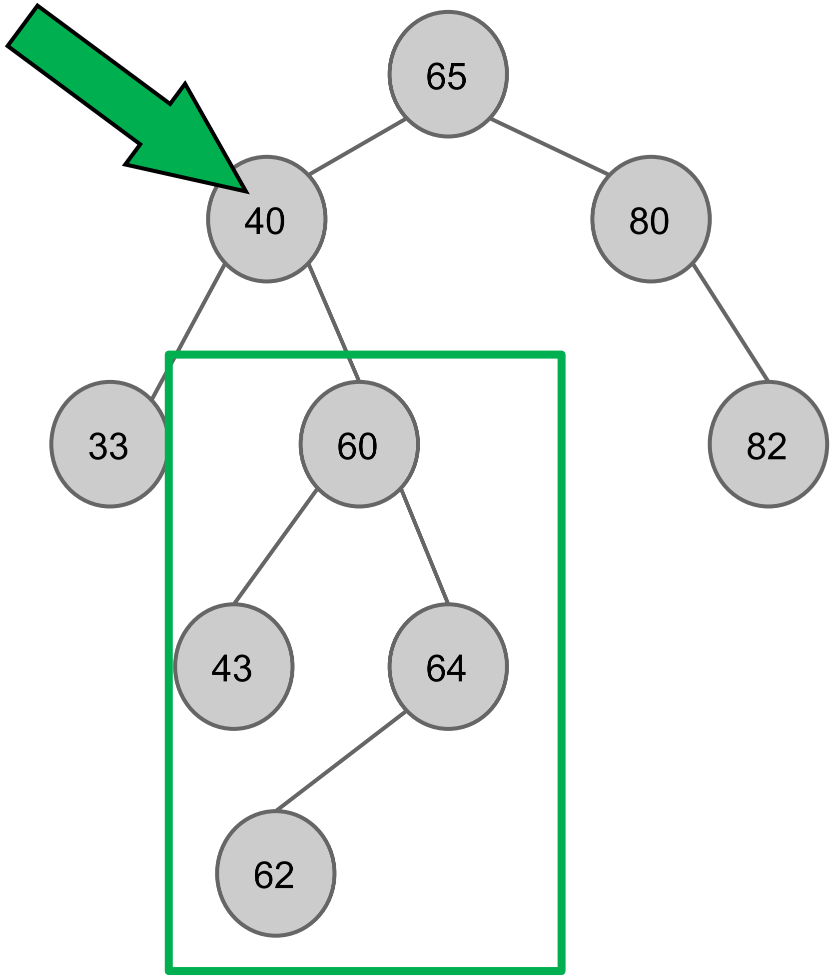
- x has a right child (easy)
- x does not have a right child (less easy)

x has a right child

`Successor(x)` must be in x's **right subtree** (the nodes **right after x** in the inorder traversal)

It's the **minimum** of x's right subtree, i.e., `TreeMinimum(x.right)`

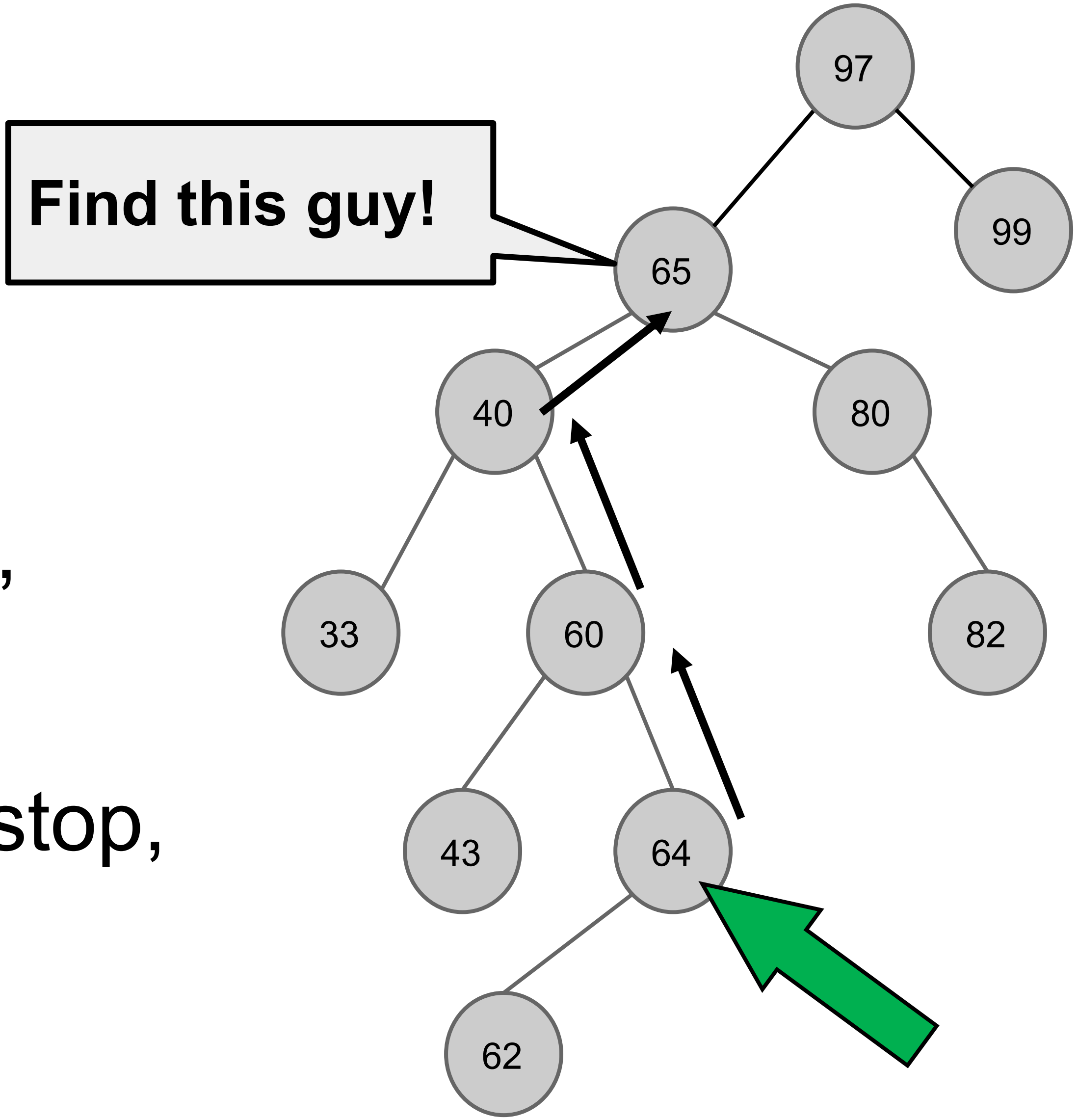
The first (smallest) node among what's right after x.



x does not have a right child

How to find

- go up to **x.p**
- if **x** is a **right** child of x.p, keep going up
- if x is a **left** child of **x.p**, stop, **x.p** is **the guy**!



x does not have a right child

Consider the **inorder traversal**

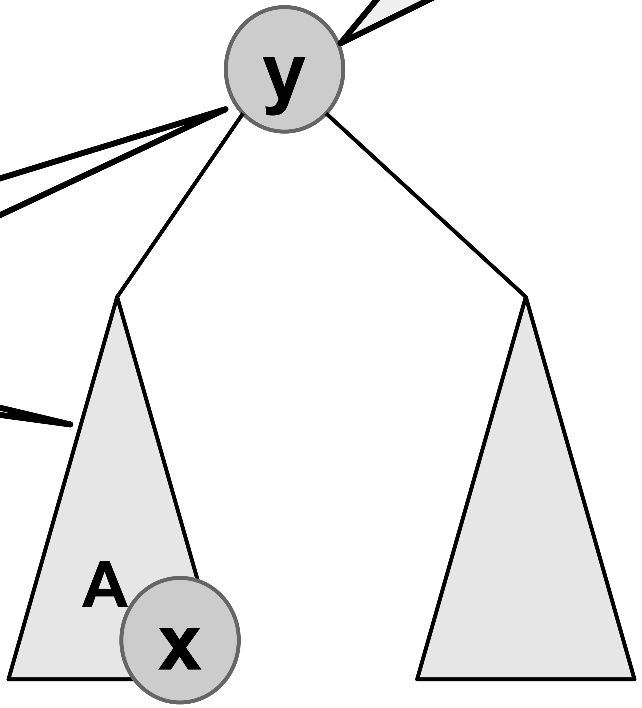
(left subtree -> root -> right subtree)

Find this guy!

x is the **last one** visited in **some node y's left subtree A** (because no right child)

The successor **y** of x is the **lowest ancestor** of **x** whose **left subtree** contains **x**

(**y** is visited right after finishing subtree **A** in inorder traversal)



Summary of the two cases of `Successor(x)`

If x has a right child

return `TreeMinimum(x.right)`

If x does not have a right child

- keep going up to x.p while x is a right child, stop when x is a left child, then return x.p
- if already gone up to the root and still not finding it, return NIL.

`Successor(x)` Pseudo-code

```
Successor(x):
    if x.right ≠ NIL:
        return TreeMinimum(x.right)
    y = x.p
    while y ≠ NIL and x == y.right: #x is right child
        x = y
        y = y.p # keep going up
    return y
```

Worst case running time

O(h), Case 1: TreeMin is O(h); Case 2: at most leaf to root

`Predecessor(x)`

works symmetrically the same way as

`Successor(x)`

Agenda Recap

ADT: **Dictionary**

Data structure: **BST**

read-only operations

```
TreeSearch(root, k)
TreeMinimum(x) / TreeMaximum(x)
Successor(x) / Predecessor(x)
```

modifying operations

```
TreeInsert(root, x)
TreeDelete(root, x)
```

7 min

`TreeInsert(root, x)`

Insert node x into the BST rooted at root

return the new root of the modified tree

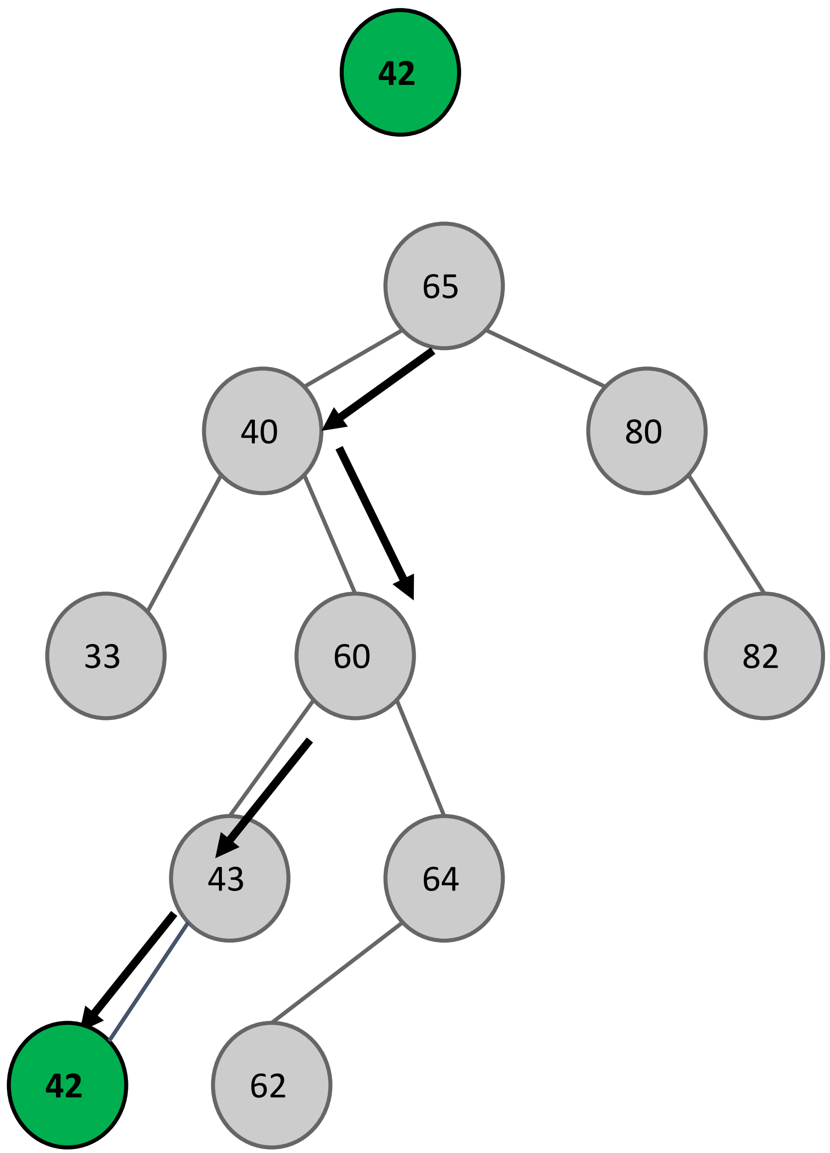
if exists y, s.t. y.key = x.key, replace y with x

TreeInsert(root, x)

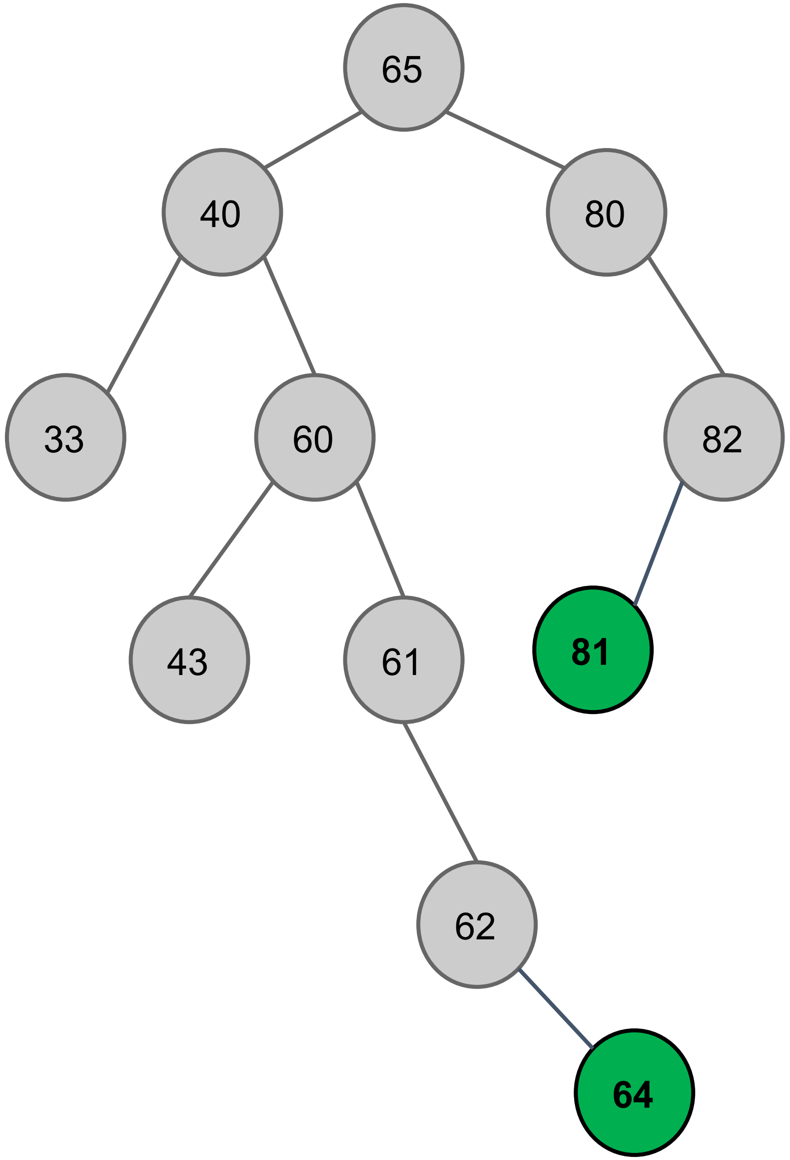
Go down, left and right like what we do in `TreeSearch`

When next position is NIL, insert there

If find equal key, replace the node

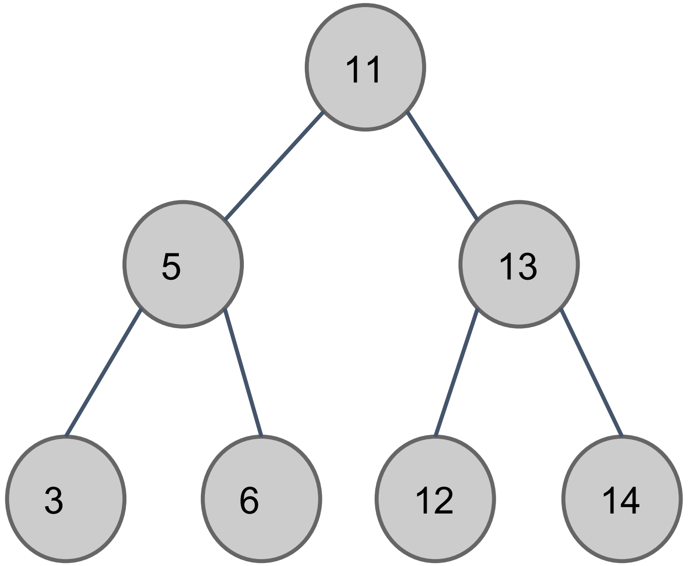


Exercise

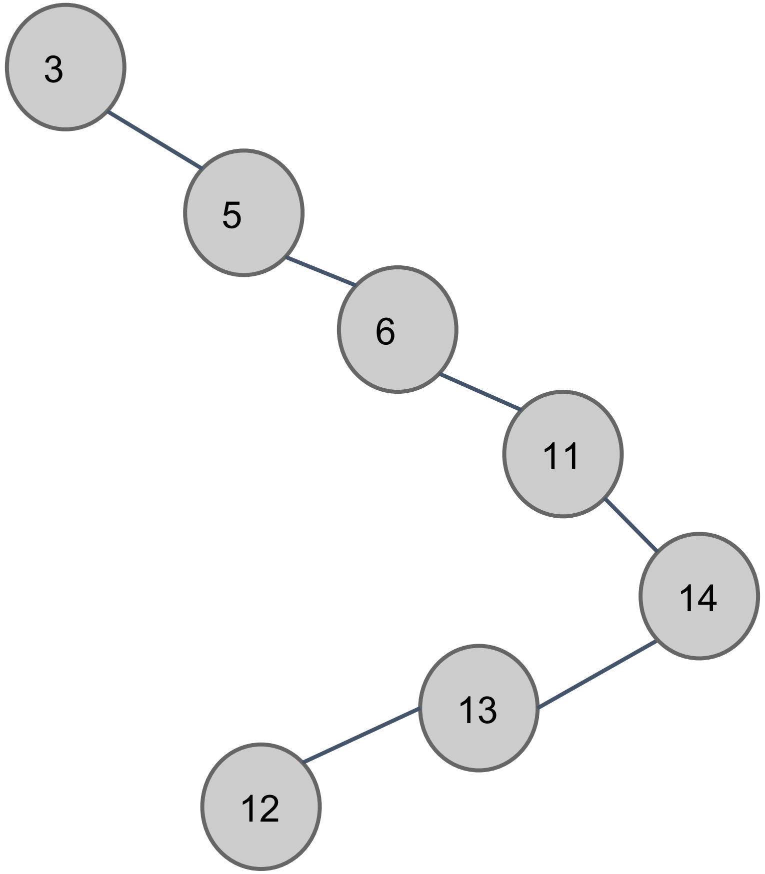


Ex 2: Insert sequence into an empty tree

Insert sequence 1:
11, 5, 13, 12, 6, 3, 14



Insert sequence 2:
3, 5, 6, 11, 14, 13, 12

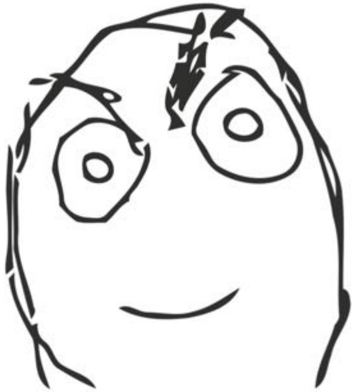


Different insert sequences result in different “shapes” (heights) of the BST.

TreeInsert(root, x): Pseudo-code

```
TreeInsert(root, x):
# insert and return the new root
    if root == NIL:
        root = x
    elif x.key < root.key:
        root.left = TreeInsert(root.left, x)
    elif x.key > root.key:
        root.right = TreeInsert(root.right, x)
    else # x.key == root.key:
        replace root with x # update x.left, x.right
    return root
```

Worst case running time:
O(h)

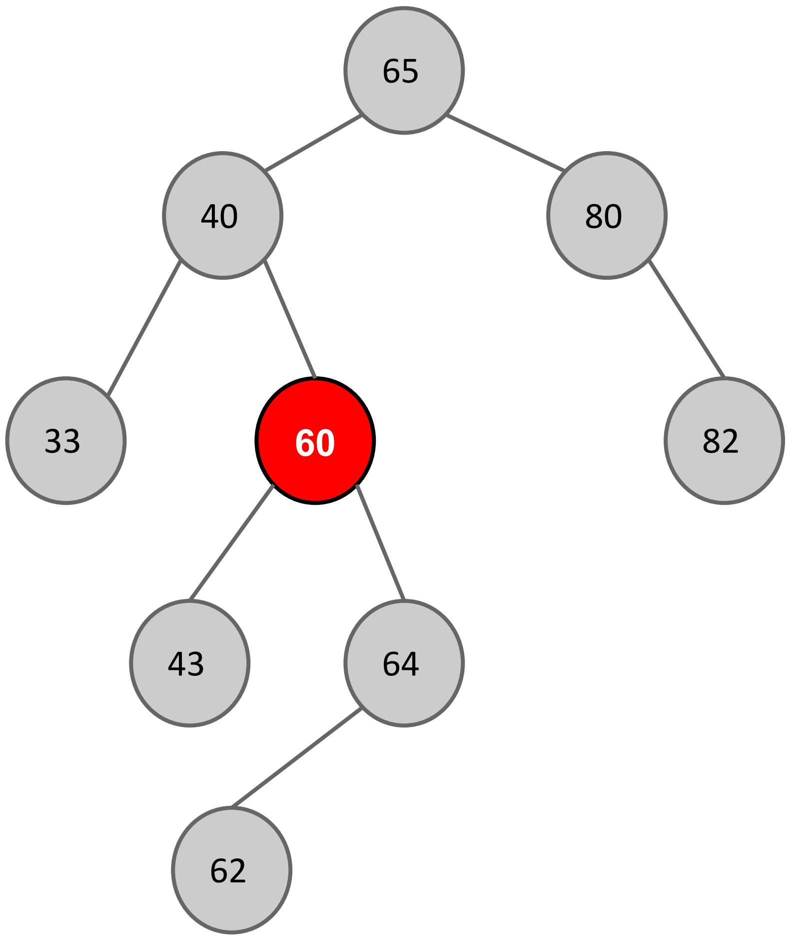


TreeDelete(root, x)

Delete node x from BST rooted at root while maintaining BST property, return the new root of the modified tree

What’s tricky about deletion?

Tree might be **disconnected** after deleting a node, need to **connect** them back together, while maintaining the **BST property**.



Delete(root, x): Organize into 3 cases

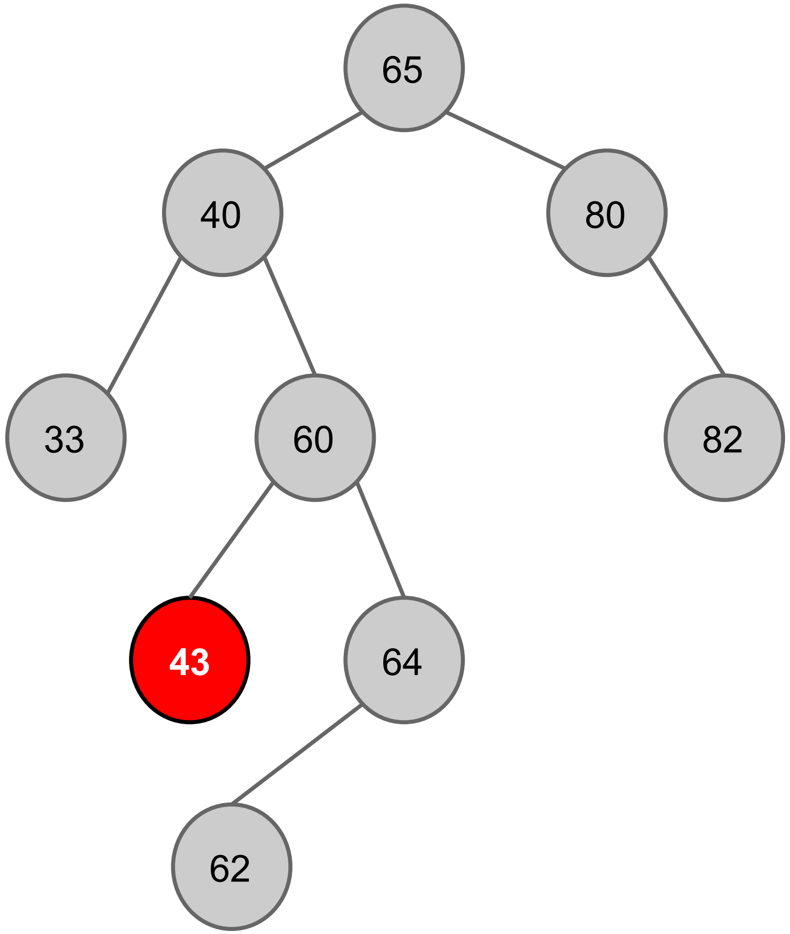
Case 1: x has **no** child — **Easy**

Case 2: x has **one** child — **Easy**

Case 3: x has **two** children — **Less easy**

Case 1: x has no child

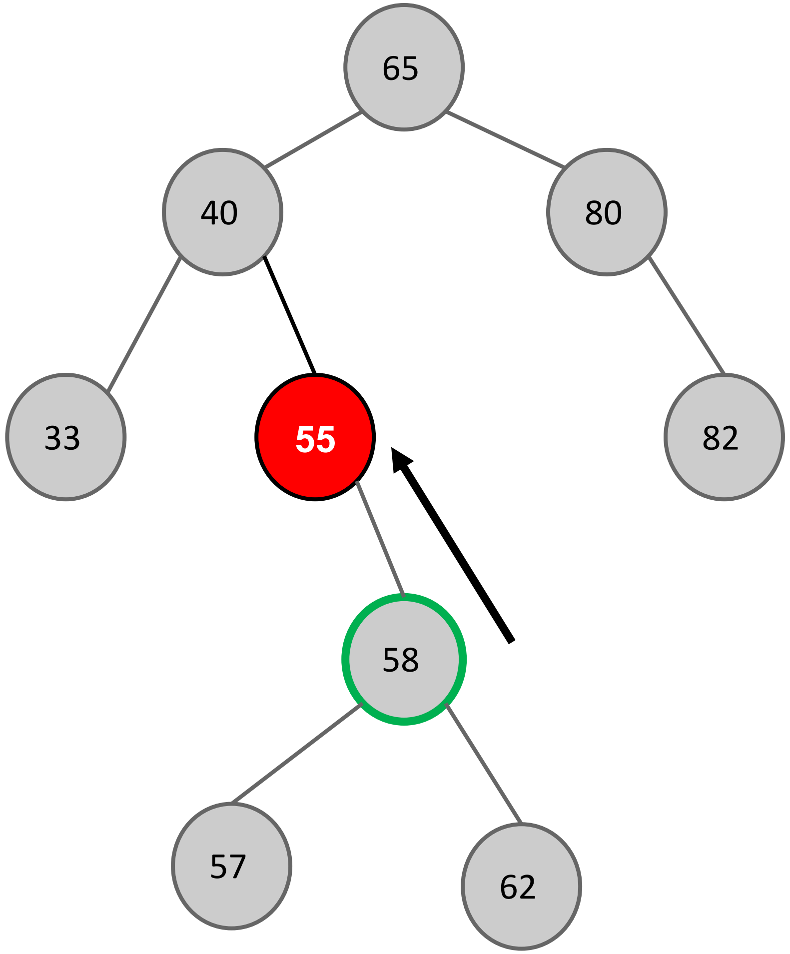
Just delete it, nothing else need to be changed.



Case 2: x has one child

First delete that node, which makes an **open spot**.

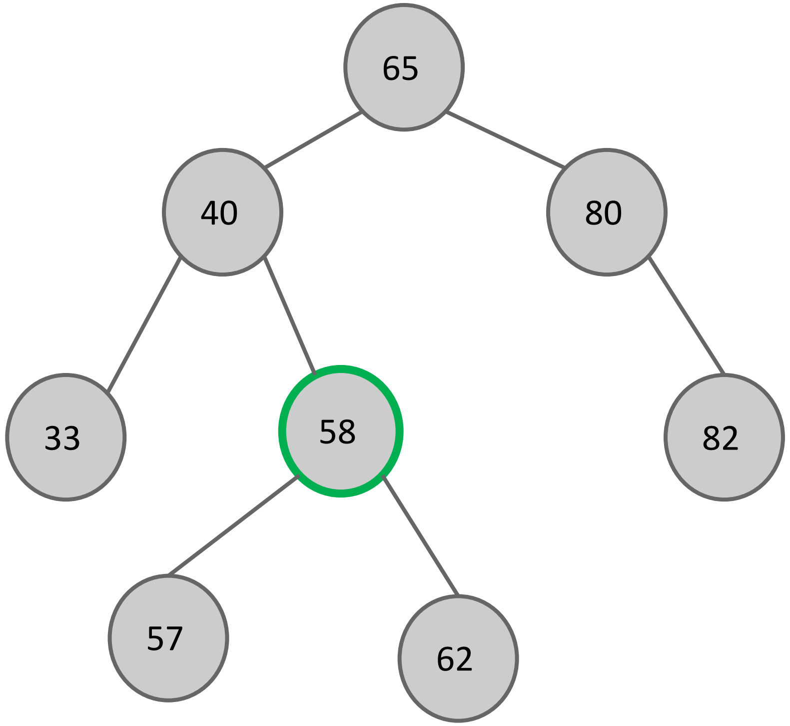
Then **promote x’s only child** to the spot, together with the only child’s subtree.



Case 2: x has one child

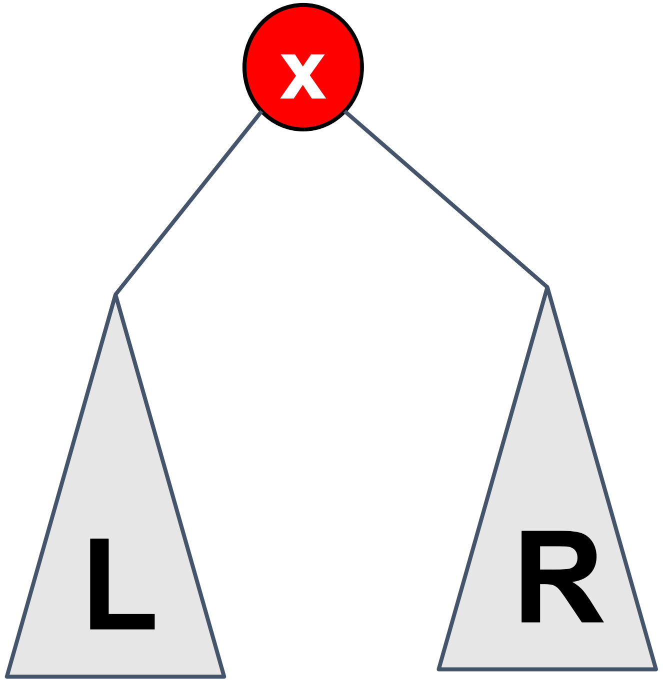
First delete that node, which makes an **open spot**.

Then promote **x's only child** to the spot, together with the only child's subtree.



Case 3: x has two children

Delete **x**, which makes an open spot.



A node **y** should fill this spot, such that **L < y < R**.

Which one should be y?

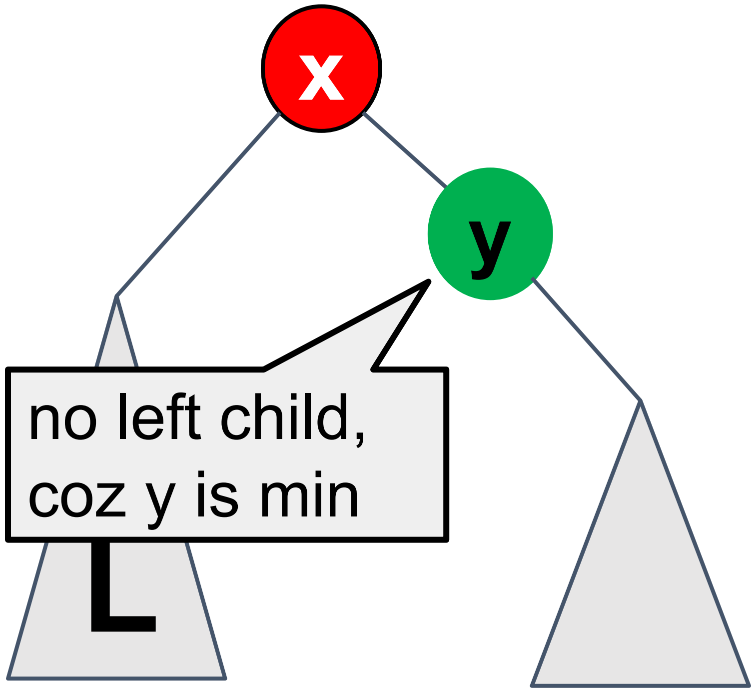
y $\leftarrow$ **the minimum of R, i.e.,** `Successor(x)`

L < y because y is in R, **y < R** because it's minimum

Further divide into two cases

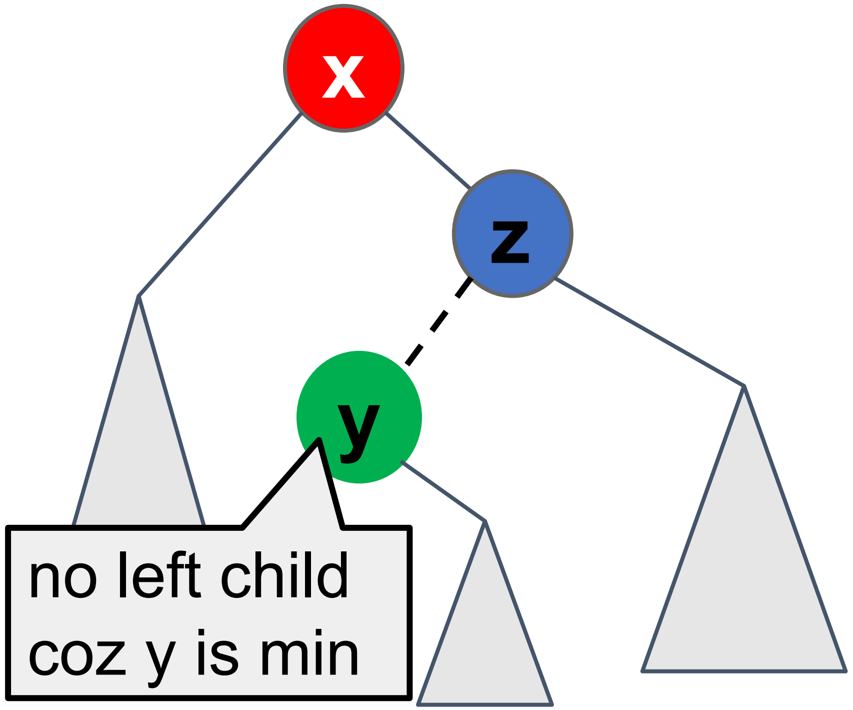
Case 3.1:

y happens to be the right child of **x**



Case 3.2:

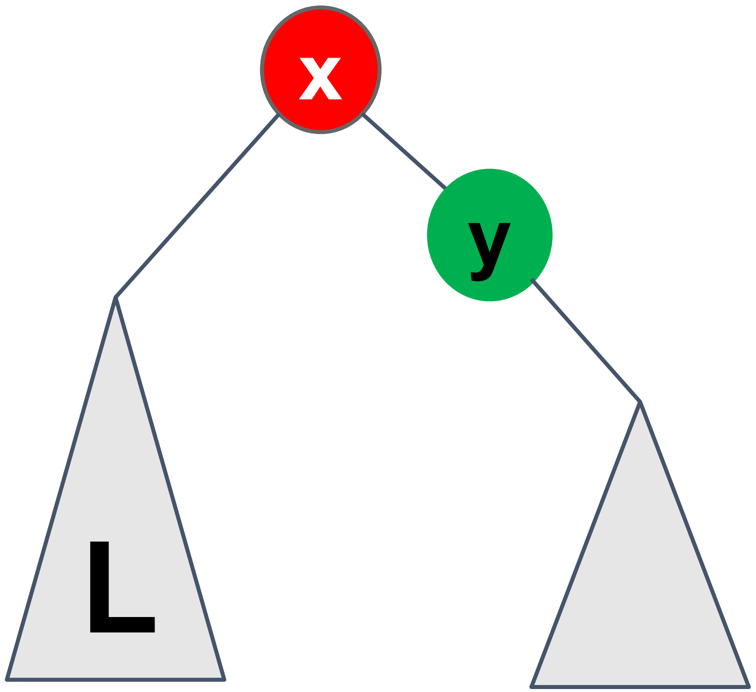
y is not the right child of **x**



Case 3.1: y is x's right child

Easy, just **promote**

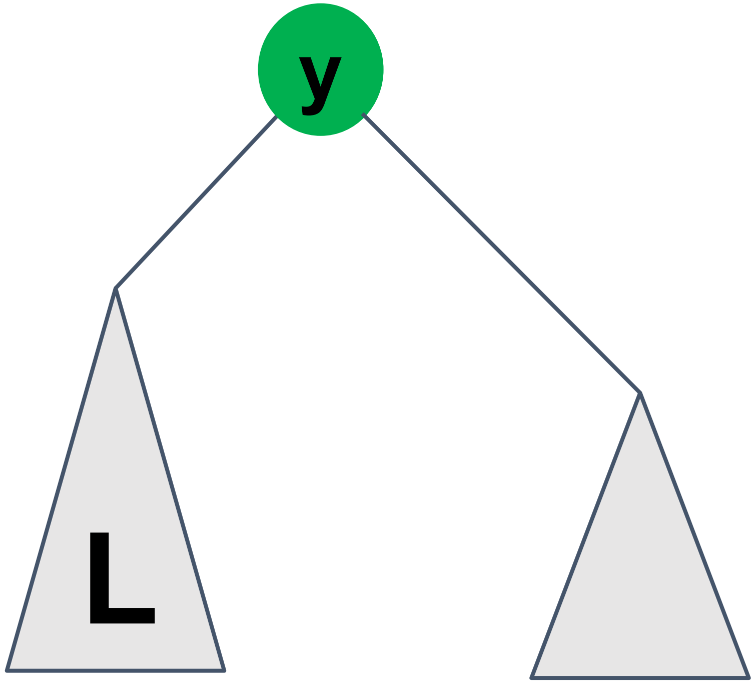
y to **x**'s spot



Case 3.1: y is x’s right child

Easy, just **promote**

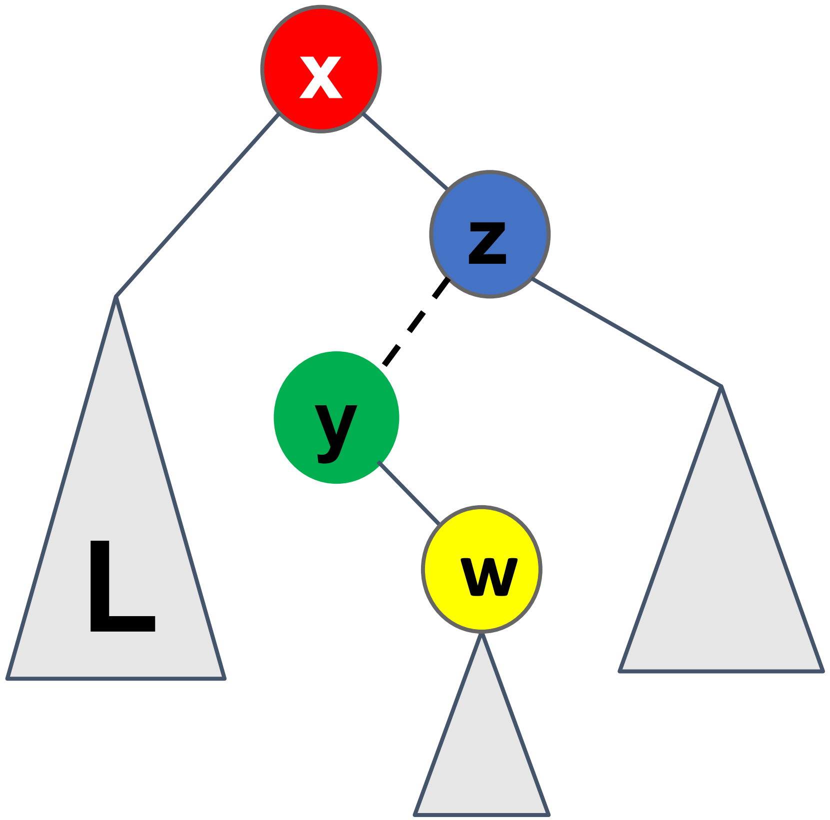
y to **x**’s spot



Case 3.2: y is NOT x’s right child

1. Promote **w** to **y**’s spot,
   y becomes free.

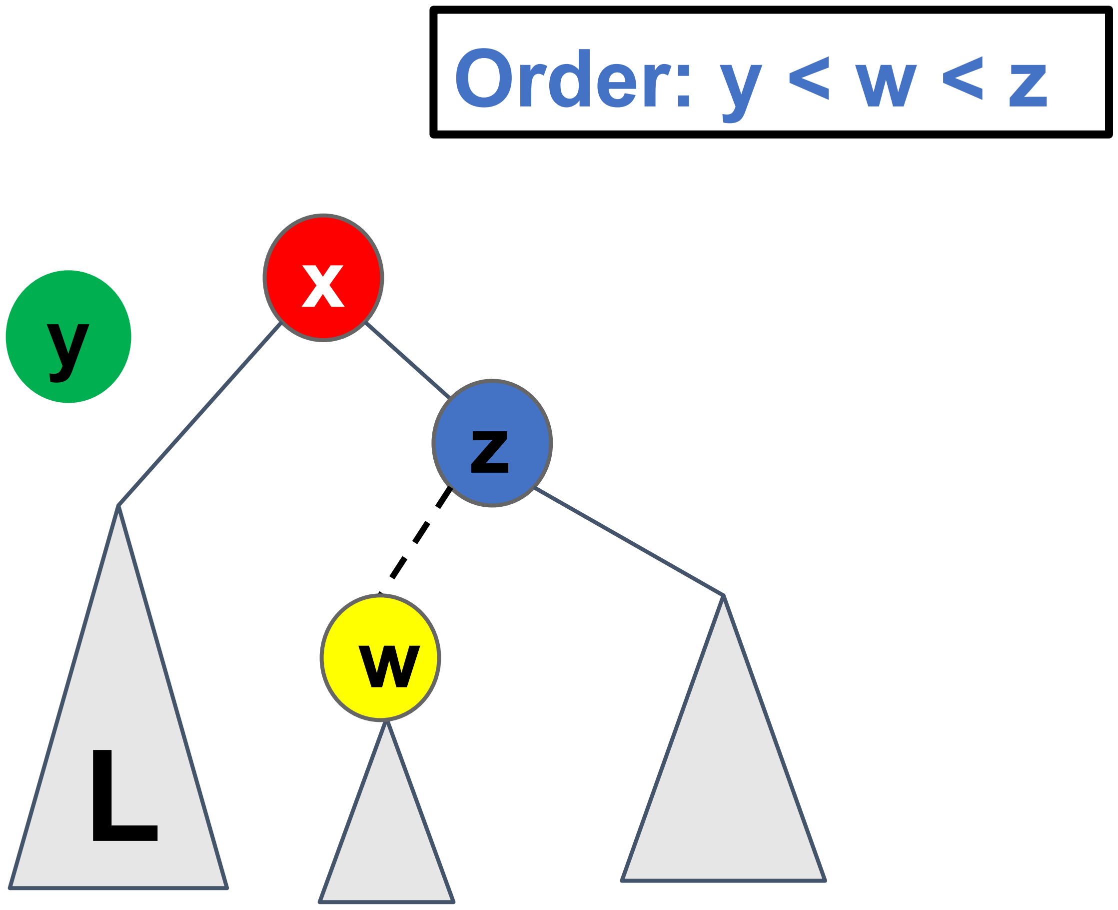
Order: y < w < z



Case 3.2: y is NOT x's right child

1. Promote **w** to **y**'s spot, **y** becomes free.

2. Make **z** be **y**'s right child (**y** adopts **z**)



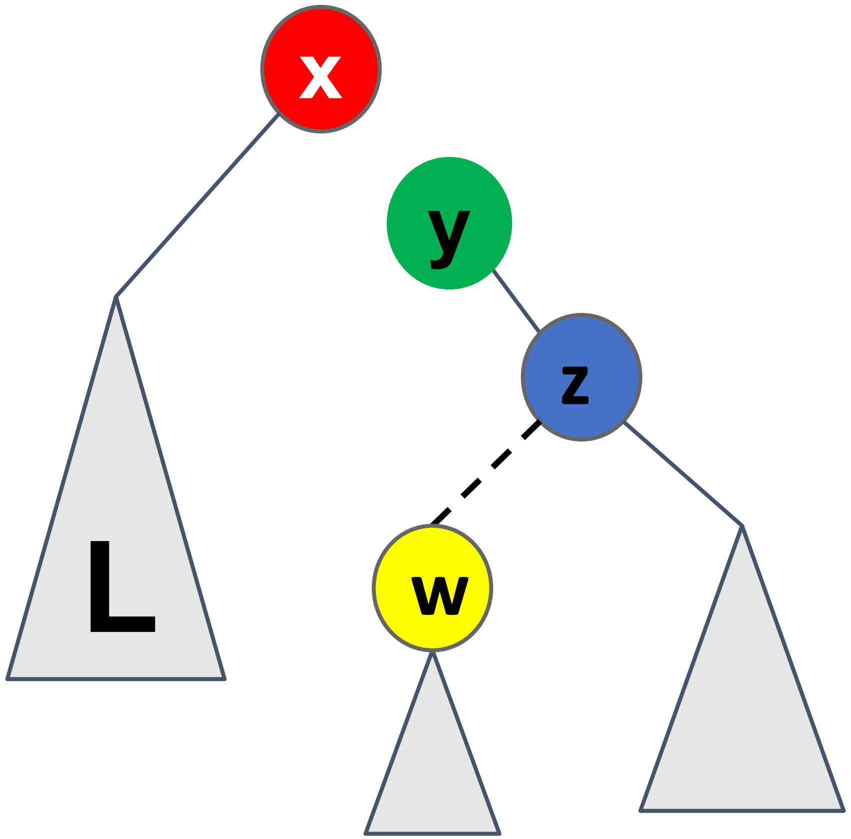
Case 3.2: y is NOT x's right child

Order: y < w < z

1. Promote **w** to **y**'s spot,
   y becomes free.

2. Make **z** be **y**'s right child
   (**y** adopts **z**)

3. Promote **y** to **x**'s spot



Case 3.2: y is NOT x's right child

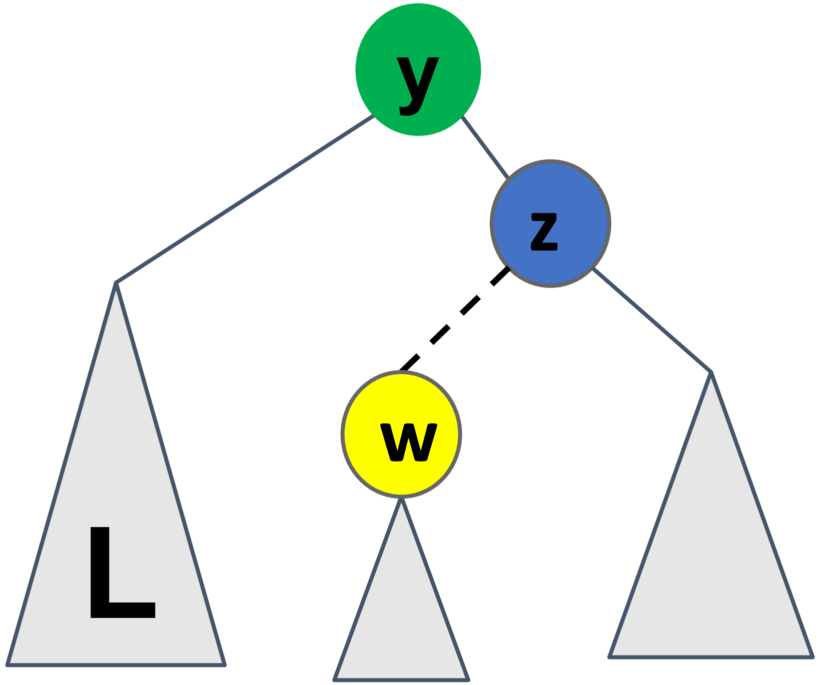
1. Promote **w** to **y**'s spot,
   y becomes free.

2. Make **z** be **y**'s right child
   (**y** adopts **z**)

3. Promote **y** to **x**'s spot

x deleted
BST order maintained, all is good.

Order: y < w < z

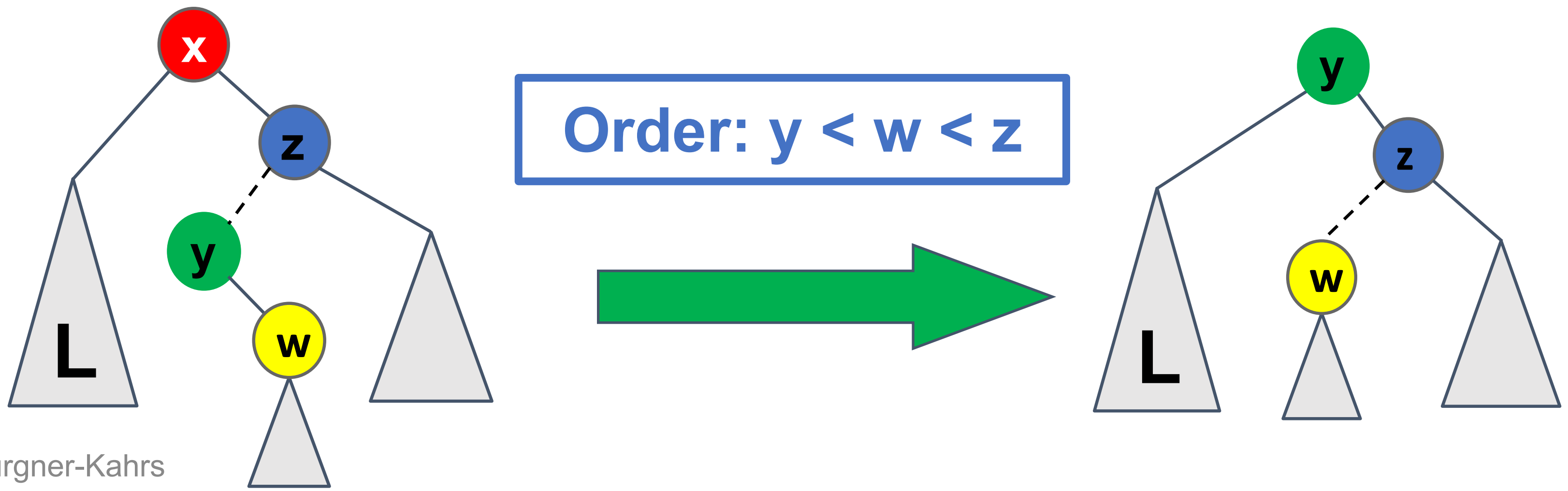


More thinking about Case 3.2

Okay, promote, adopt, promote... I can see that it works, pretty clever.

But **HOW ON EARTH** can I **come up** with this kind of clever algorithms!

Thinking Process: **understand** the BST property (the **invariant**), **predict** the final shape of the tree, and see **how to get there**.



Summarize `TreeDelete(root, x)`

Case 1: x has no child, **just delete**

Case 2: x has one child, **promote**

Case 3: x has two children, y = `Successor(x)`

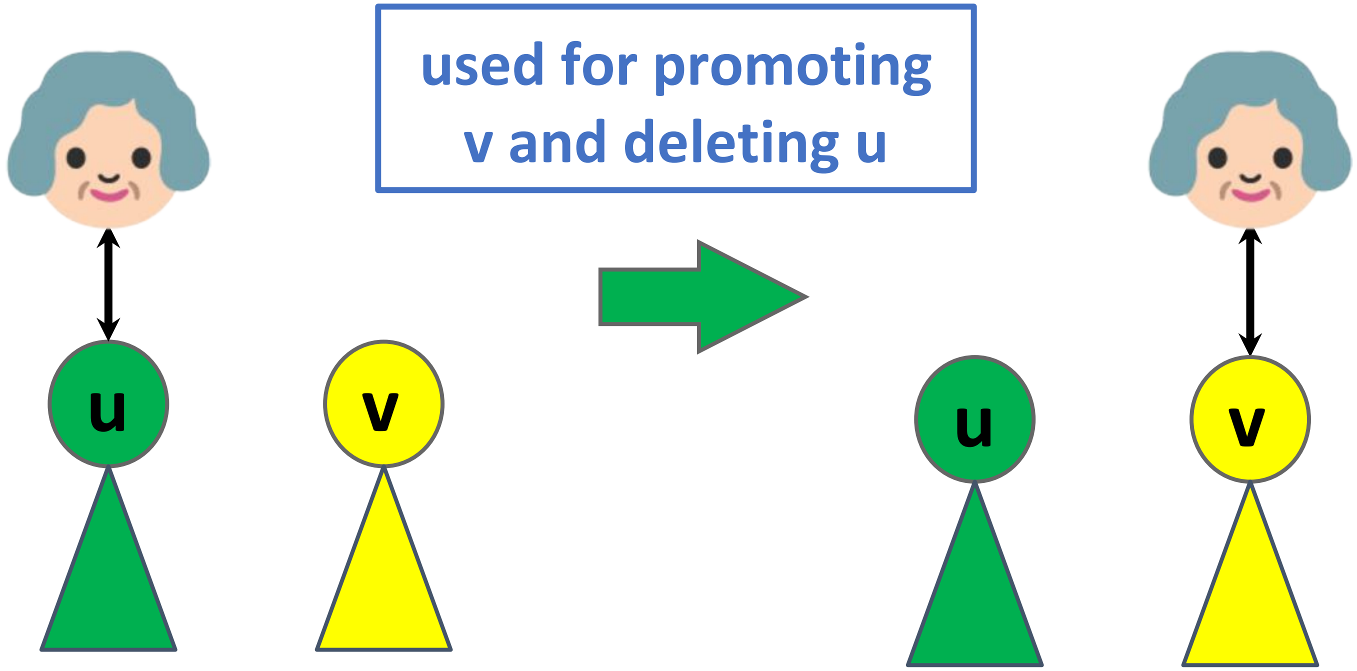
- **Case 3.1**: y is x's right child, **promote**
- **Case 3.2**: y is NOT x's right child
  - **promote** y's right child
  - y **adopt** x's right child
  - **promote** y

`TreeDelete(root,x)` Pseudo-code

CLRS Chapter 12.3

Key: Understand `Transplant(root,u,v)`

 # v takes away u’s parent



```
Transplant(root, u, v):
# v takes away u's parent
 if u.p == NIL: # if u is root
    root = v   # v replaces u as root
 elif u == u.p.left:# if u is parent's left child
    u.p.left = v #parent accepts v as left child
 else: # if u is parent's right child
    u.p.right = v #parent accepts v as right child
 if v ≠ NIL:
    v.p = u.p # v accepts new parent
```

Case 1 & 2

Case 3

Case 3.2

```
TreeDelete(root, x):
  if x.left == NIL:
      Transplant(root, x, x.right)
  elif x.right == NIL:
      Transplant(root, x, x.left)
  else:
      y = TreeMinimum(x.right)
      if y.p ≠ x:
          Transplant(root, y, y.right)
          y.right = x.right
          y.right.p = y
      Transplant(root, x, y)
      y.left = x.left
      y.left.p = y
  return root
```

Promote right child

Promote left child

get Successor(x)

y is not right child of x

promote **w**

y adopts **z**

promote y

update pointers

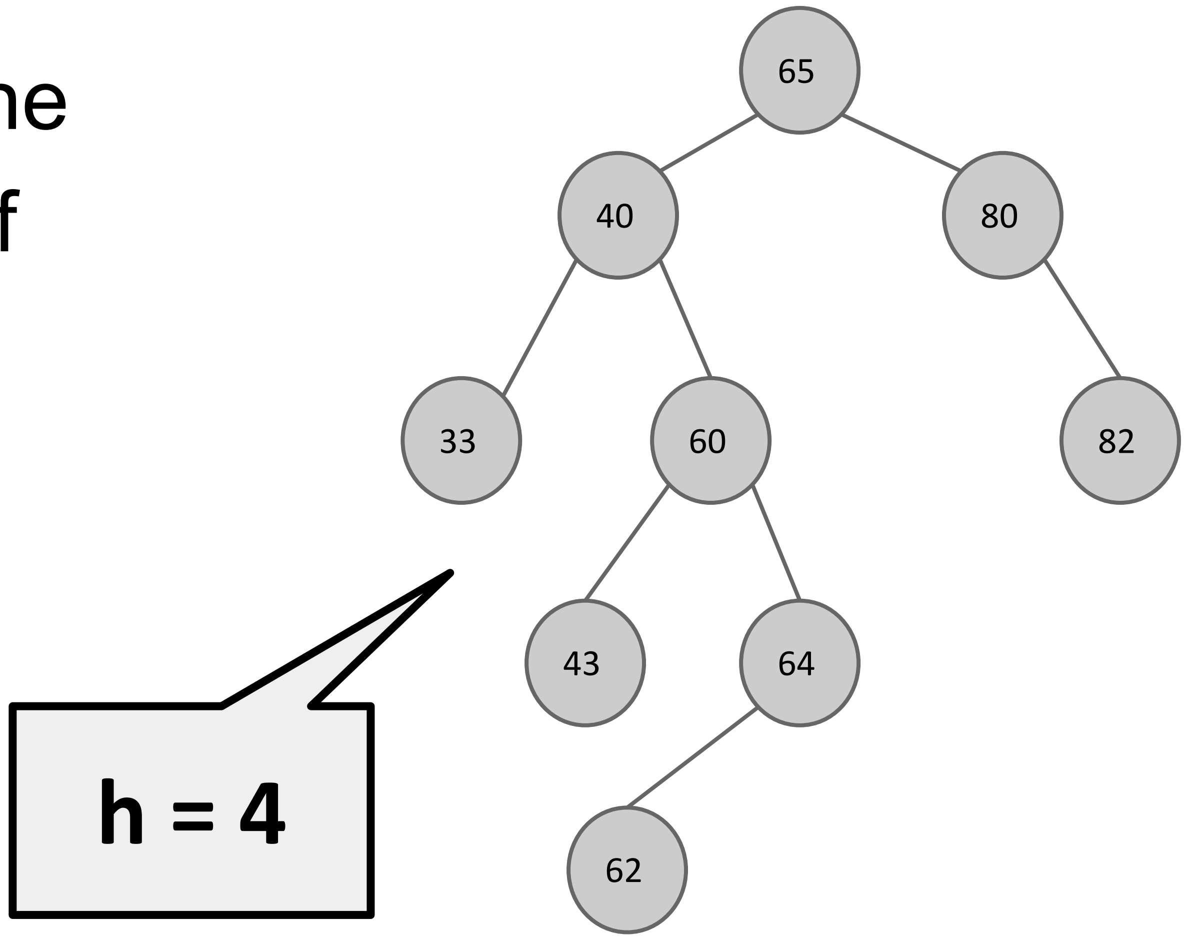
TreeDelete(root, x) worst case running time

O(h) (time spent on TreeMinimum)

Now, about that h (height of tree)

Definition: Height of a tree

The longest path from the root to a leaf, in terms of number of **edges**.



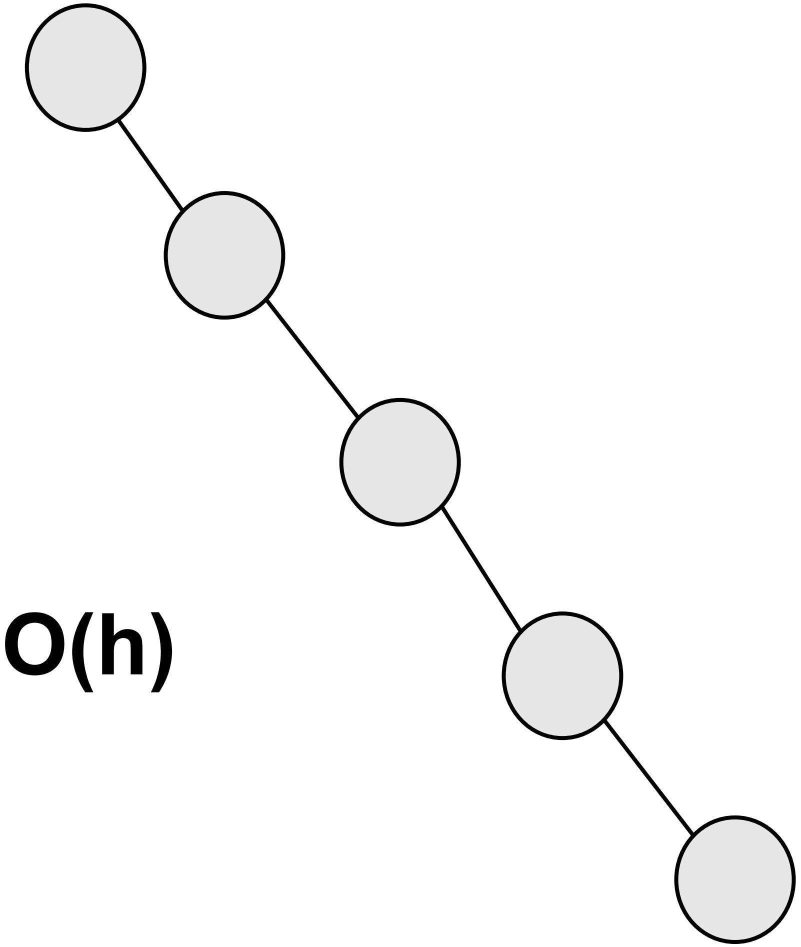
Consider a BST with n nodes, what's the highest it can be?

h = n-1

i.e, in worst case

h ∈ Θ(n)

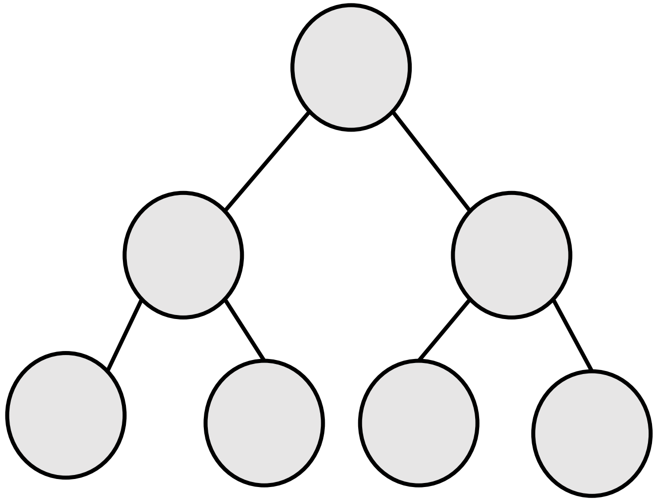
so all the operations we learned with **O(h)** runtime, they are **O(n)** in worst case



So, what's the best case for h ?

In best case, $h \in \Theta(\log n)$

A **Balanced BST** guarantees to have height in $\Theta(\log n)$



Therefore, all the **O(h)** become **O(log n)**

Recap Quiz

Question 1

How to visit the nodes in a BST in a sorted order?

- ❑ inorder traversal
- ❑ preorder traversal
- ❑ postorder traversal
- ❑ level-by-level traversal

Question 2

Node x has two children. The predecessor of x is the tree __________ in x's __________ subtree.

- ❑ maximum, right
- ❑ maximum, left
- ❑ minimum, left
- ❑ minimum, right

Question 3

Insert three keys into a BST in this order: 2, 6, 3.
What is the height of the resulting tree?

- ❑ 0
- ❑ 1
- ❑ 2
- ❑ 3

Next week

A Balanced BST called **AVL tree**