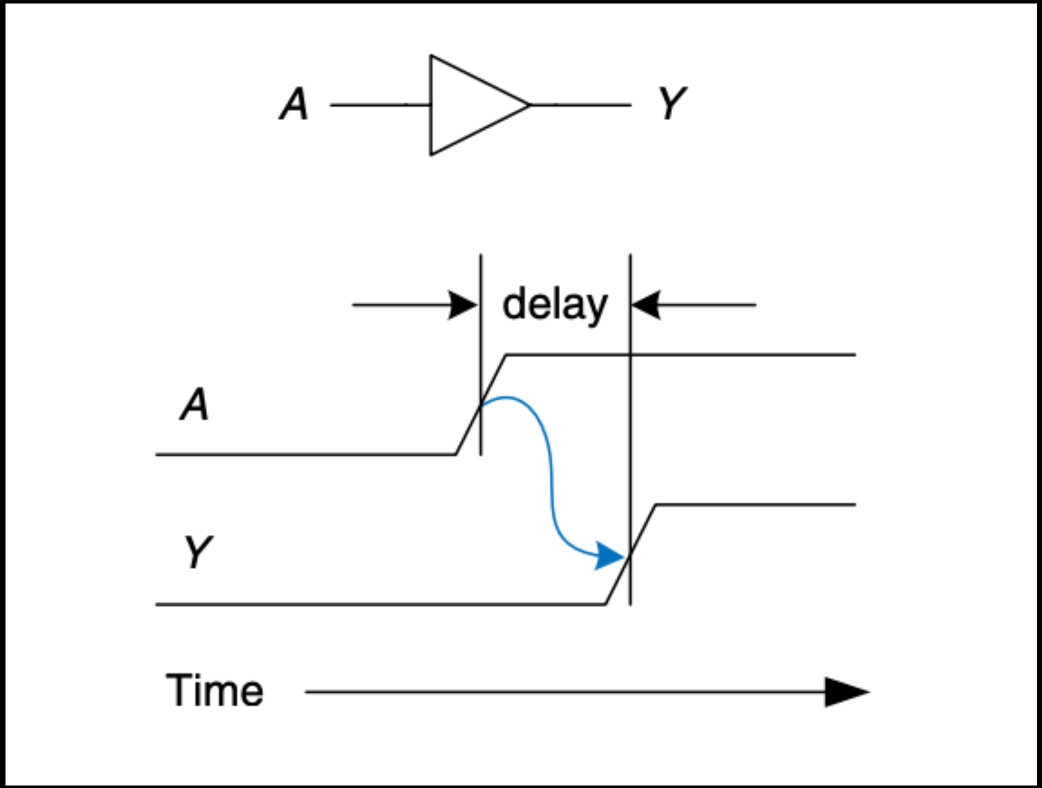
CSC258 Week 6

Circuit Timing

Timing

- So far we have been worrying whether a circuit is correct.
- Now let's think about how to make a circuit fast.
- Key concept: latency
  - propagation delay
  - contamination delay

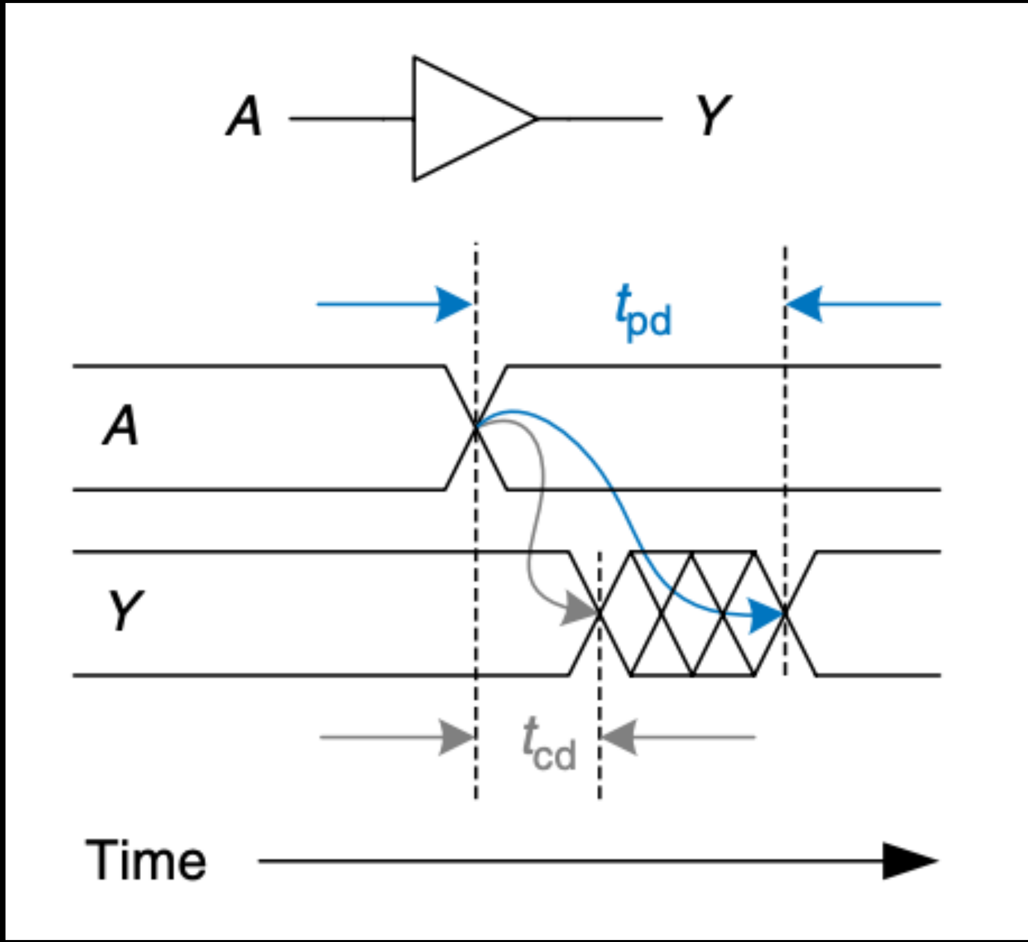
Delay Example



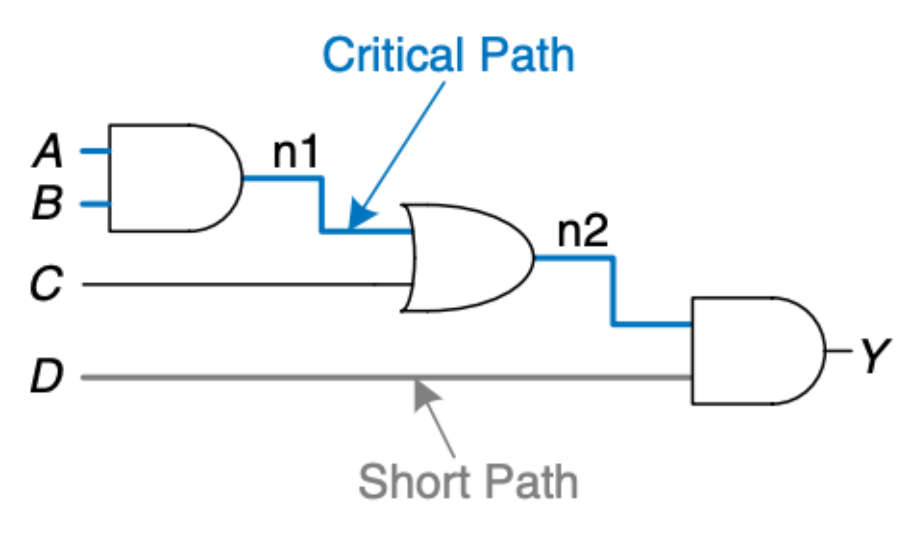
- We measure the interval between the two “50% points” of the changing signals

Propagation & Contamination Delay

- **Propagation delay**: the maximum time from when an input changes until the output or outputs reach their final value.
- **Contamination delay**: the minimum time from when an input changes until any output starts to change its value.

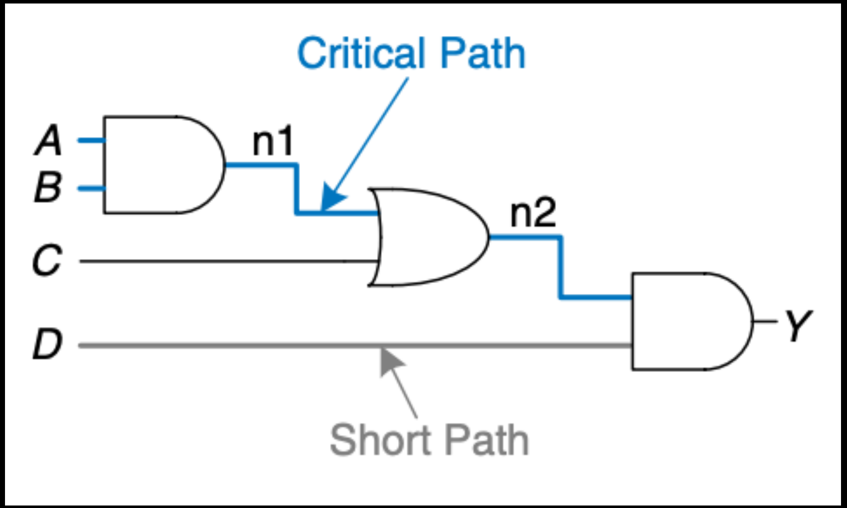


Given a circuit diagram,
calculate its propagation delay
and contamination delay



Need to know

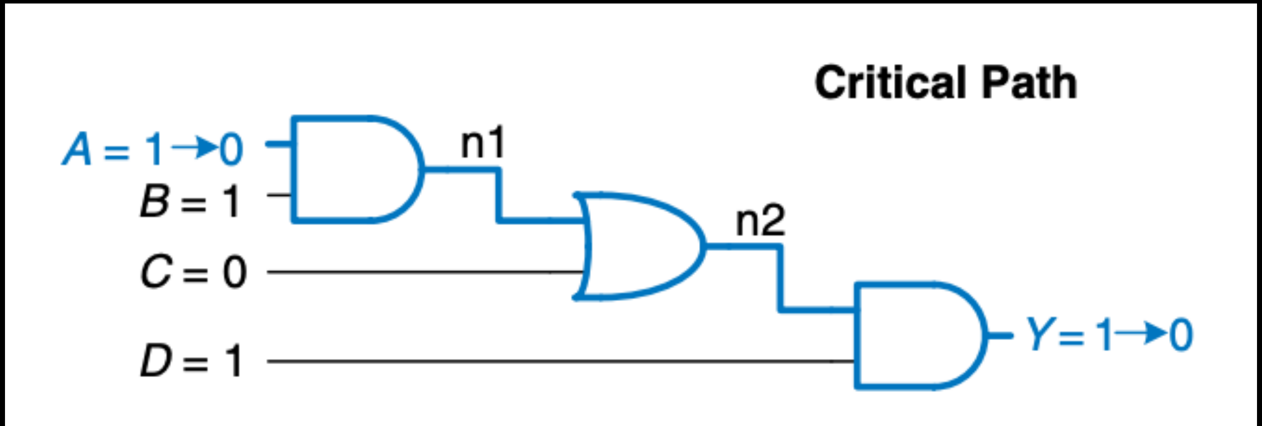
- The propagation and contamination delay of each logic gate used



Gate	t_pd (propagation)	t_cd (contamination)
2-input AND	100 picoseconds	60 picoseconds
2-input OR	120 picoseconds	40 picoseconds

Calculate Propagation Delay

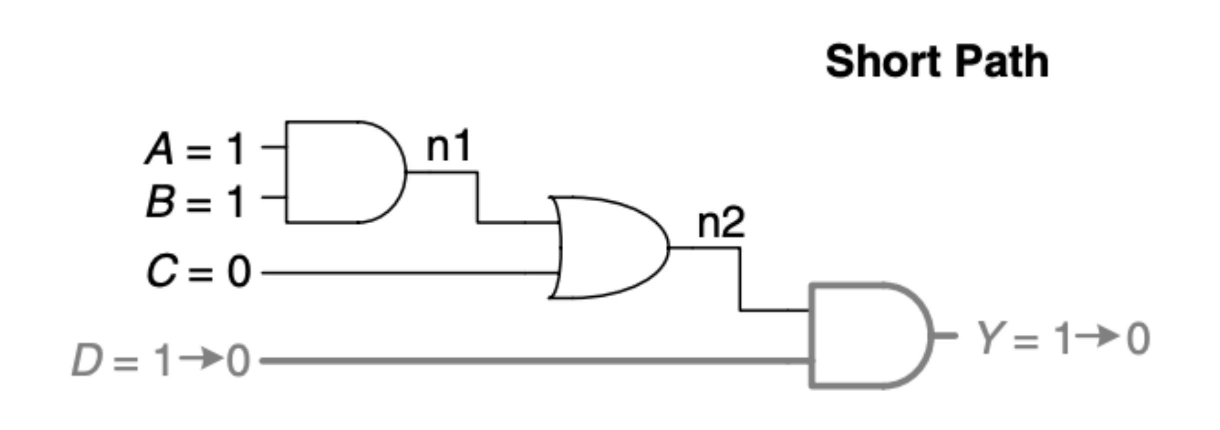
- Find the **critical path** (path with the largest number of gates)
- then sum up the **propagation** delay of all the gates on the critical path
- 100 + 120 + 100 = 320 picoseconds



Gate	t_pd	t_cd
2-input AND	100 ps	60 ps
2-input OR	120 ps	40 ps

Calculate Contamination Delay

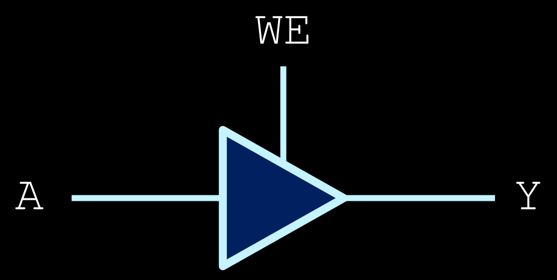
- Find the **short path** (path with the smallest number of gates)
- then sum up the **contamination** delay of all the gates on the short path
- 60 seconds



Gate	t_pd	t_cd
2-input AND	100 ps	60 ps
2-input OR	120 ps	40 ps

Knowing how to calculate delays allows us to design circuits that are fast.

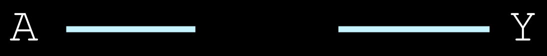
Quick intro:
Tri-state buffer



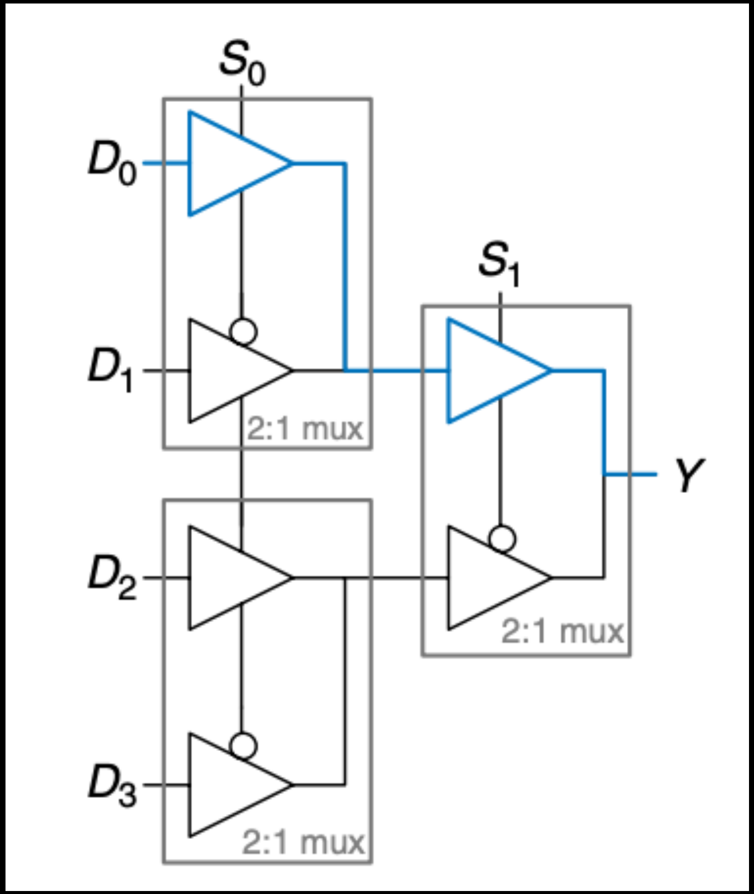
WE	A	Y
0	x	z
1	0	0
1	1	1

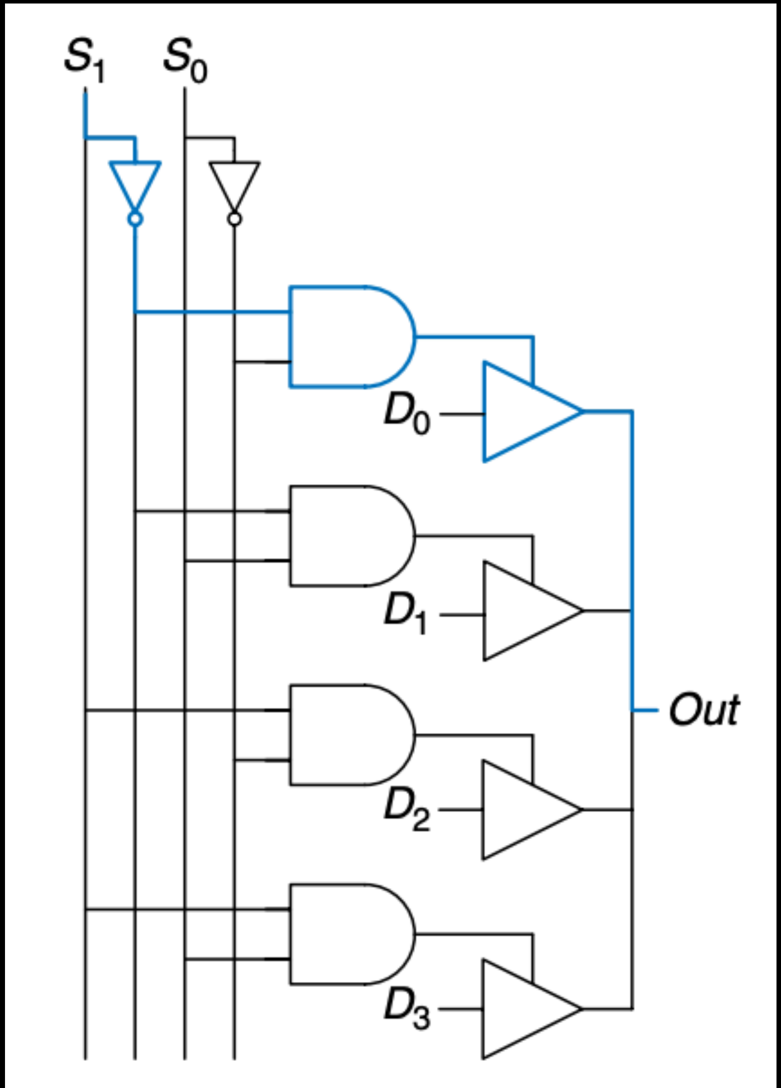
WE = 1

WE = 0



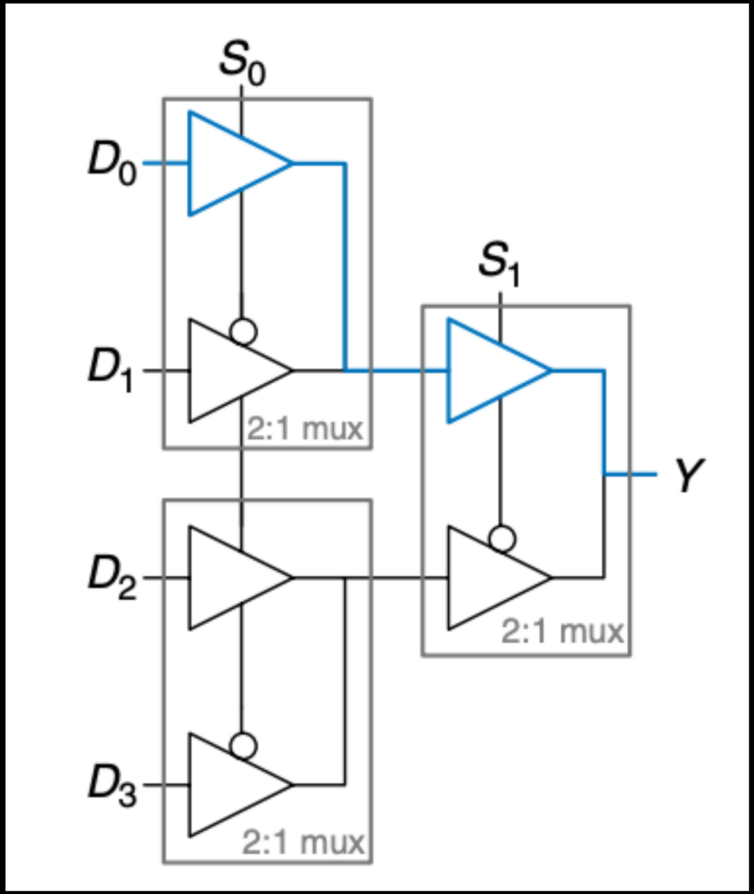
Example: design fast circuit

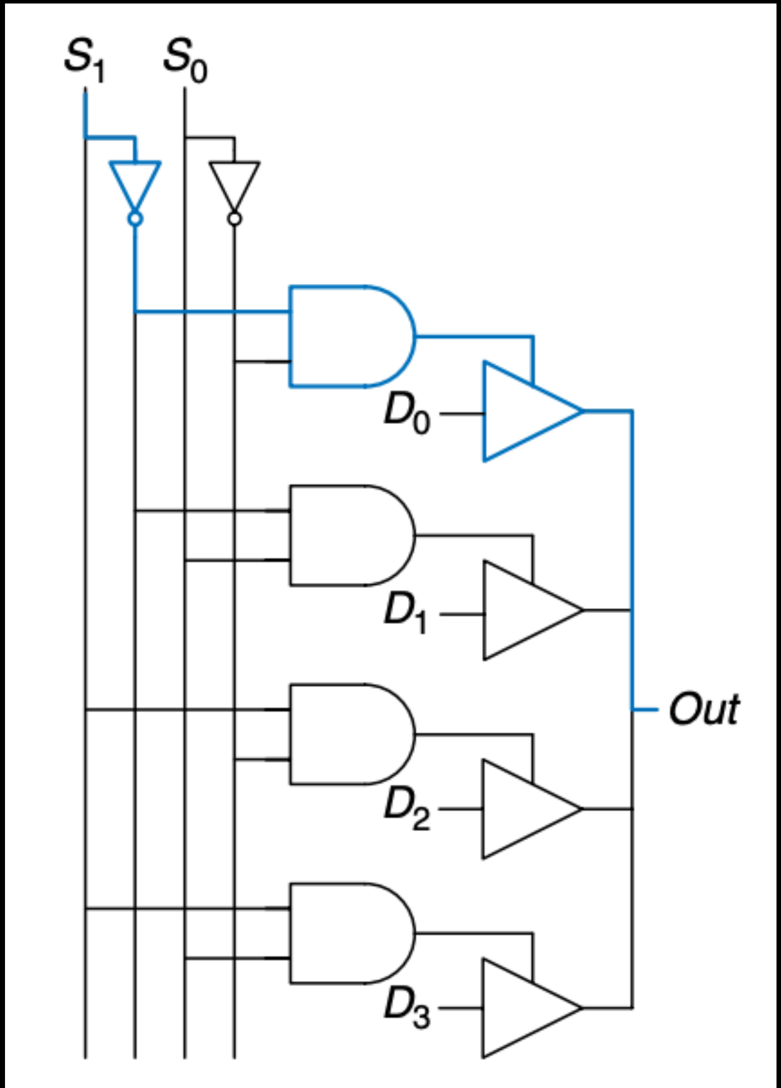




4-1 muxes

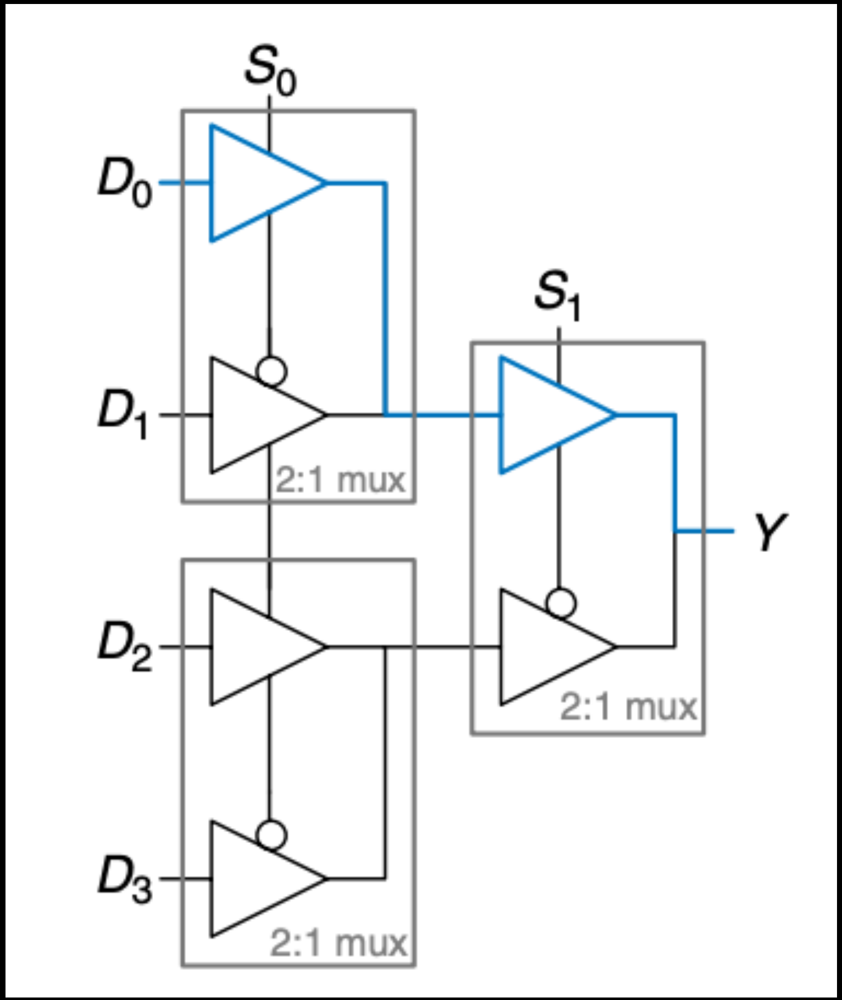
Example: design fast circuit





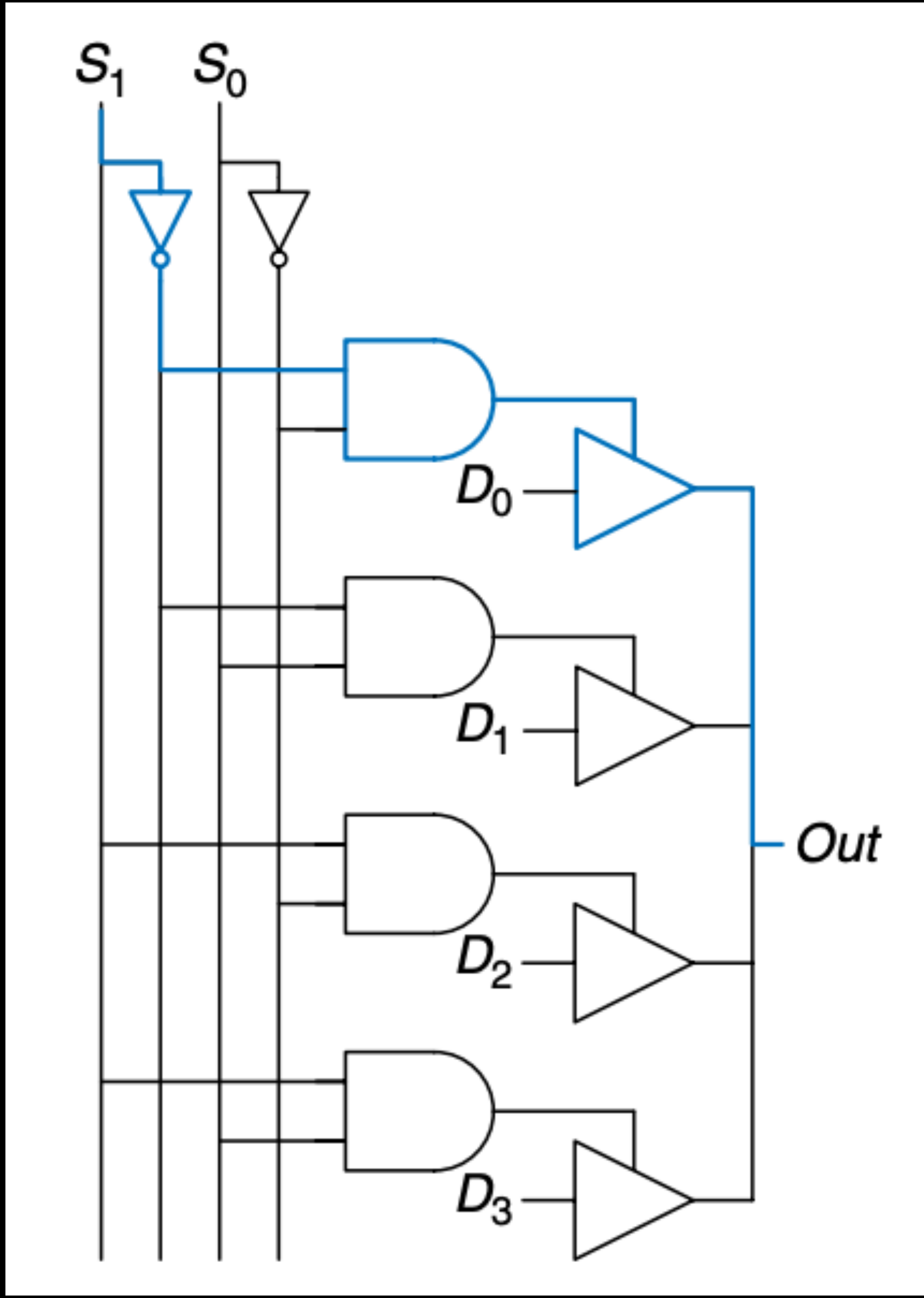
- We care about the **propagation** delays of the two circuits.
  - it tells us "how soon I can get the answer"
- More specifically, we care about the D-to-Y delay and S-to-Y delay because D and S may arrive at different time.

Gate	t_{pd} (ps)
NOT	30
2-input AND	60
3-input AND	80
4-input OR	90
tristate (A to Y)	50
tristate (enable to Y)	35



- D-to-Y propagation delay:
- 2 x TRISTATE_AY = 100

- S-to-Y propagation delay
- TRISTATE_ENY + TRISTATE_AY
- = 35 + 50 = 85



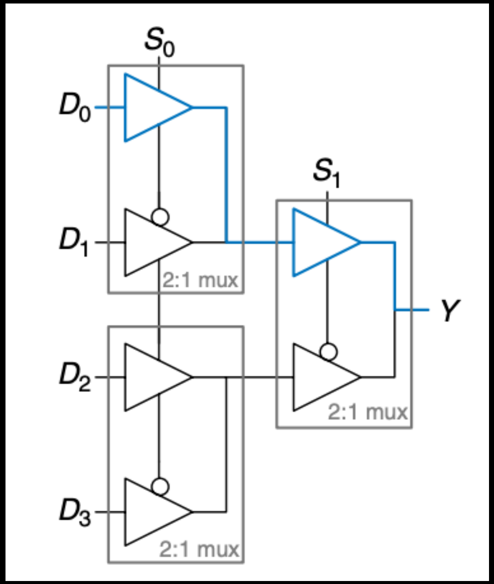
Gate	t_{pd} (ps)
NOT	30
2-input AND	60
3-input AND	80
4-input OR	90
tristate (A to Y)	50
tristate (enable to Y)	35

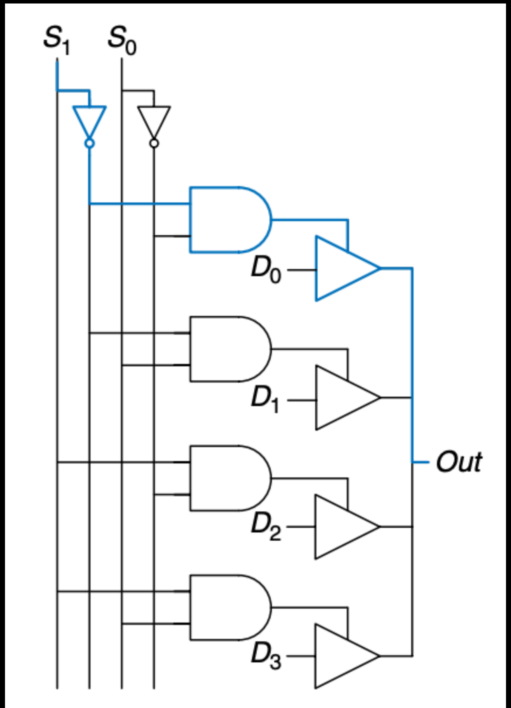
- D-to-Y propagation delay:
- TRISTATE_AY = 50

- S-to-Y propagation delay
- NOT + AND2 + TRISTATE_ENY
- = 30 + 60 + 35 = 125

Analysis result

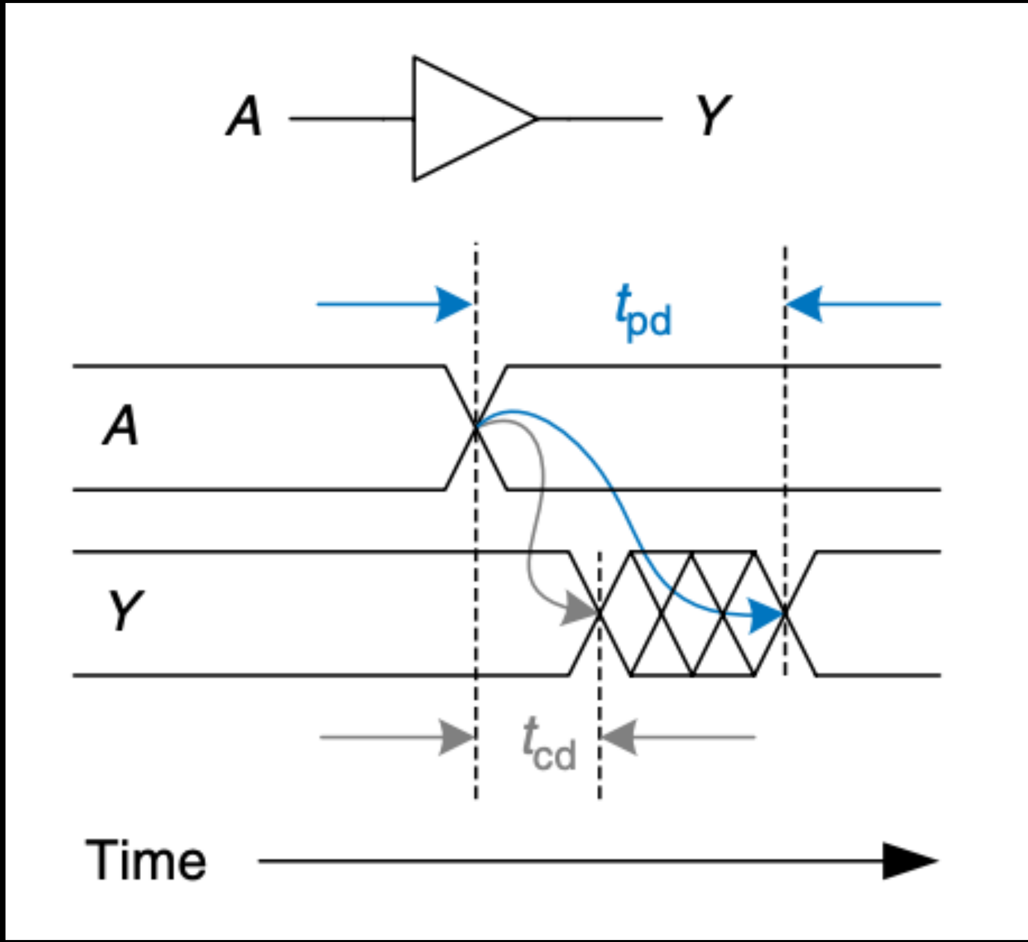
- Circuit 1 propagation:
  - D-to-Y: 100 ps
  - S-to-Y: 85 ps
- Circuit 2 propagation
  - D-to-Y: 50 ps
  - S-to-Y: 125 ps
- Which circuit is faster?
  - What if D and S arrive at the same time?
  - What if D arrives earlier than S?
  - What if S arrives earlier than D?

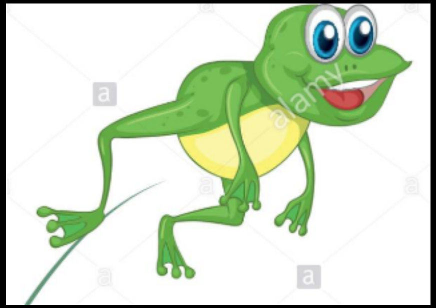




Delays: the lower/higher, the better?

- Propagation delay, typically, should be upper-bounded.
  - shorter propagation means getting answer faster
  - How to make it lower?
  - shorten the critical path
- Contamination delay, typically, should be lower-bounded
  - want to reliably sample the value before change.
  - How to make it longer?
  - add **buffers** to the short path



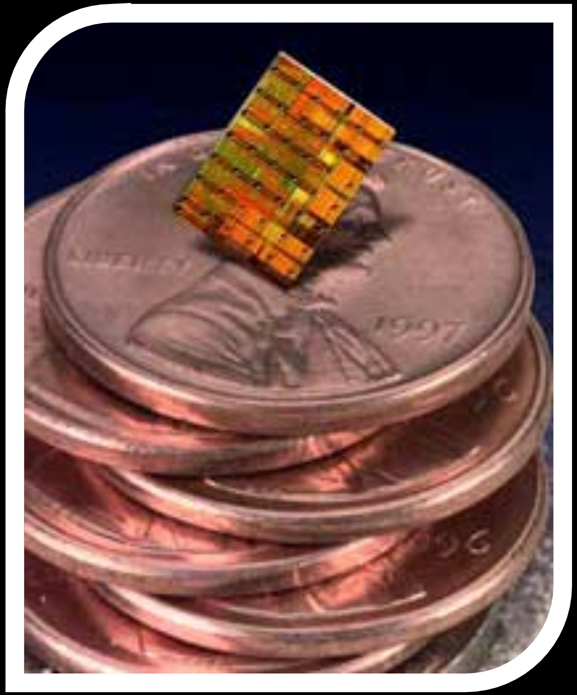


New Topic:

Processor Components

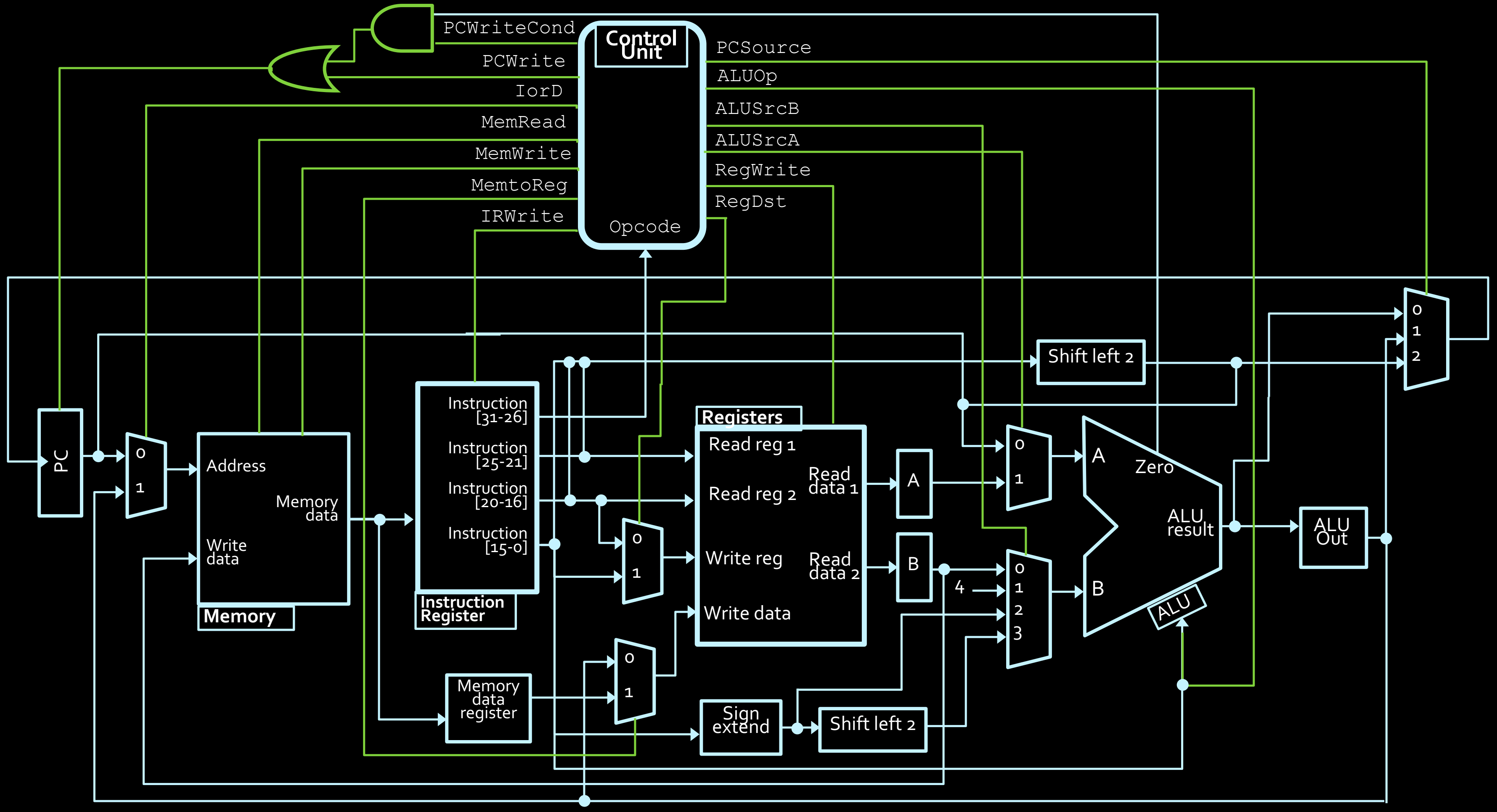
Using what we have learned so far (combinational logic, devices, sequential circuits, FSMs), how do we build a processor?

Microprocessors



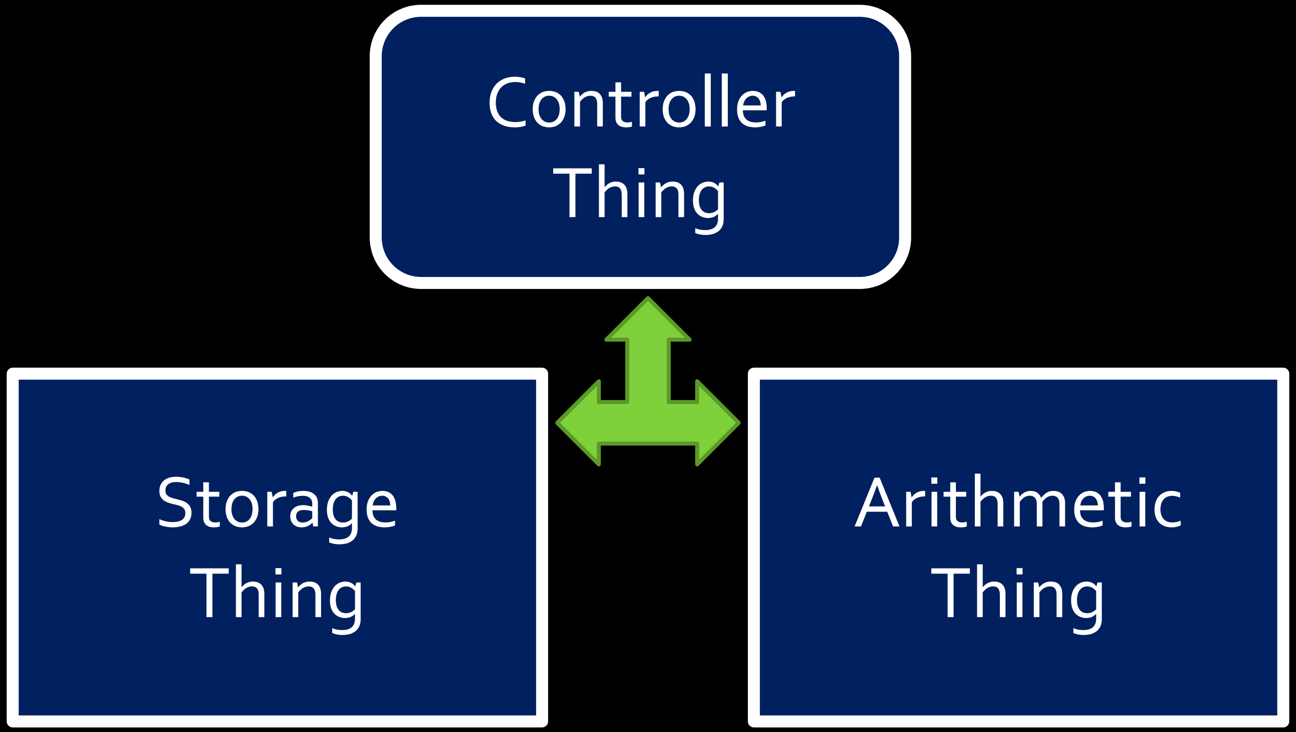
- So far, we've been talking about making devices, such as adders, counters and registers.
- The ultimate goal is to make a microprocessor, which is a digital device that processes input, can store values and produces output, according to a set of on-board instructions.

The Final Destination



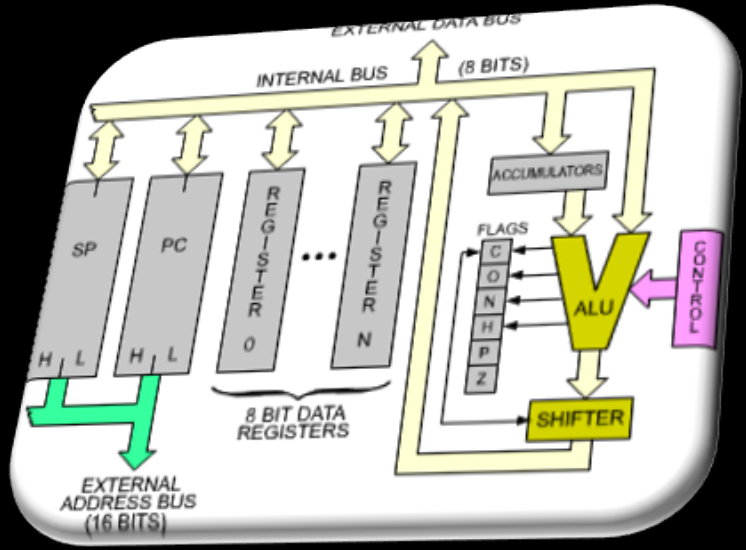
Deconstructing processors

- Processors aren't so bad when you consider them piece by piece:



Microprocessors

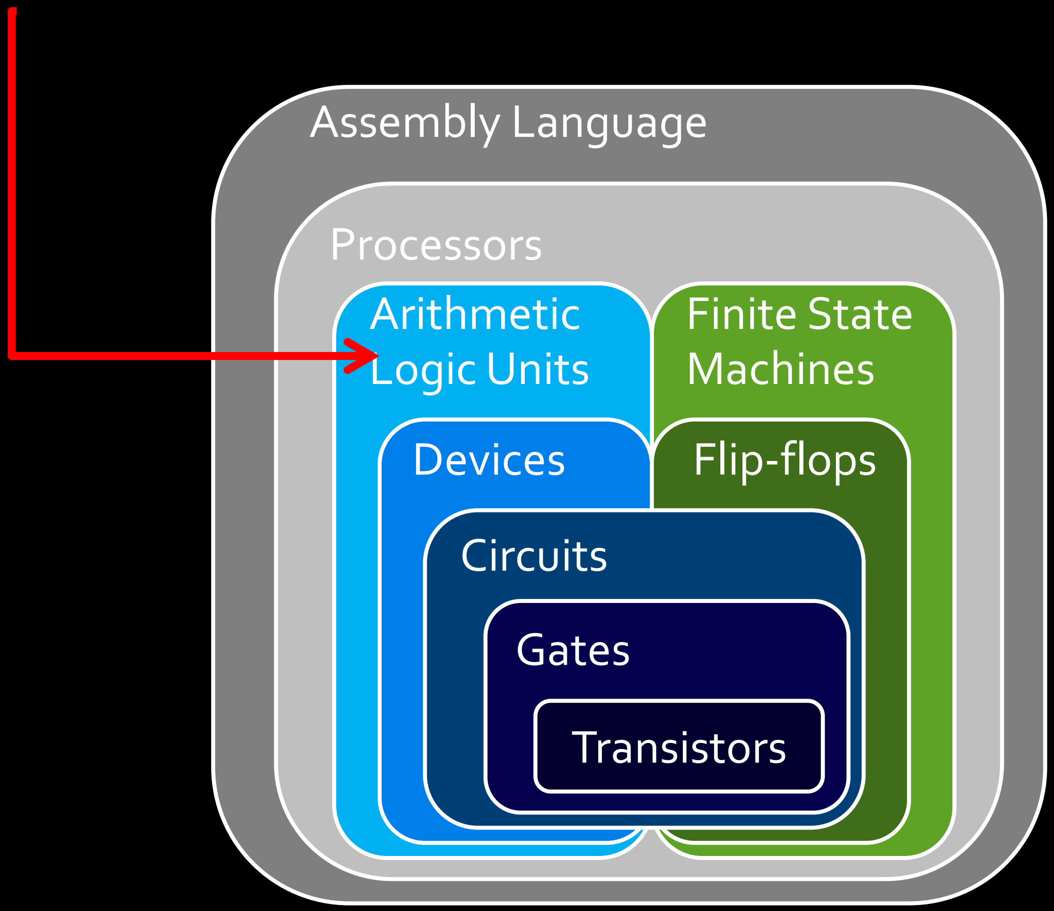
- These devices are a combination of the units that we've discussed so far:
  - Registers to store values.
  - Adders and shifters to process data.
  - Finite state machines to control the process.
- Microprocessors are the basis of all computing since the 1970's, and can be found in nearly every sort of electronics.



The "Arithmetic Thing"

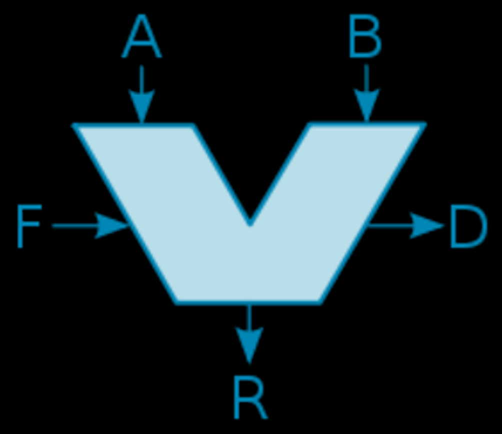
aka: the Arithmetic Logic Unit (ALU)

We are here


Assembly Language
Processors
Arithmetic Logic Units
Finite State Machines
Devices
Flip-flops
Circuits
Gates
Transistors

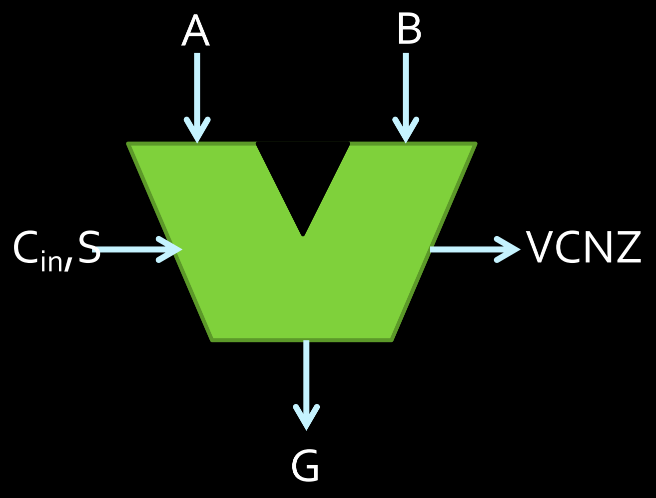
Arithmetic Logic Unit

- The first microprocessor applications were calculators.
  - Recall the unit on adders and subtractors.
  - These are part of a larger structure called the arithmetic logic unit (ALU).
- This larger structure is responsible for the processing of all data values in a basic CPU.



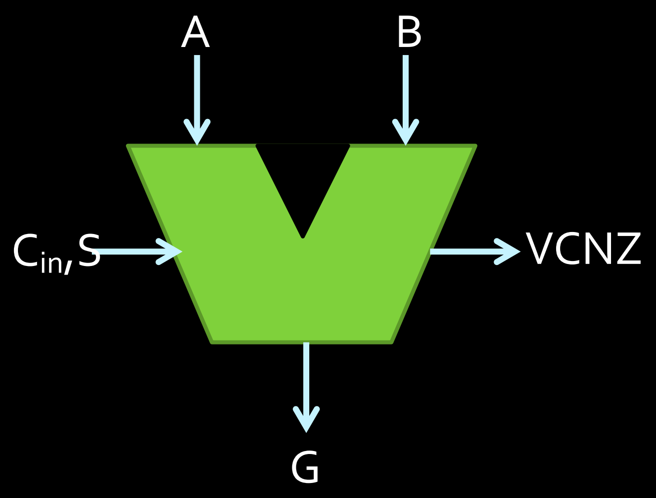
ALU inputs

- The ALU performs all of the arithmetic operations covered in this course so far, and logical operations as well (AND, OR, NOT, etc.)
  - A and B are the oprands
  - The select bits (S) indicate which operation is being performed (S2 is a mode select bit, indicating whether the ALU is in arithmetic or logic mode).
  - The carry bit C_{in} is used in operations such as incrementing an input value or the overall result.



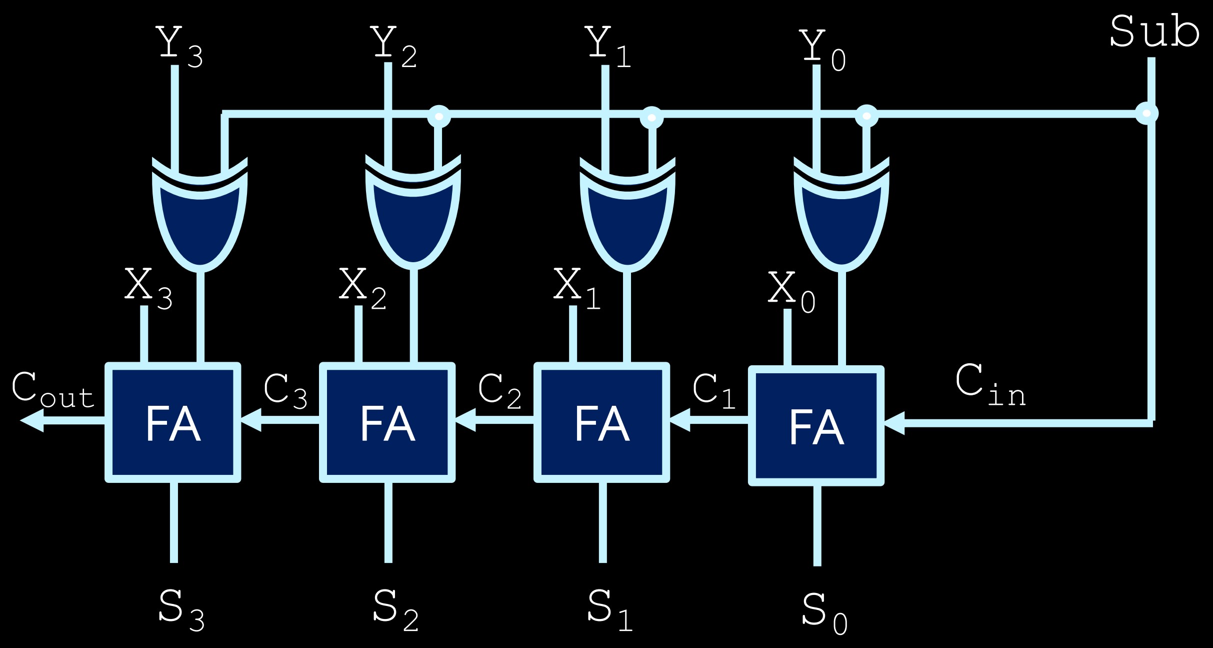
ALU outputs

- In addition to the input signals, there are output signals V, C, N & Z which indicate special conditions in the arithmetic result:
  - V: overflow condition
    - The result of the operation could not be stored in the n bits of G, meaning that the result is incorrect.
  - C: carry-out bit
  - N: Negative indicator
  - Z: Zero-condition indicator



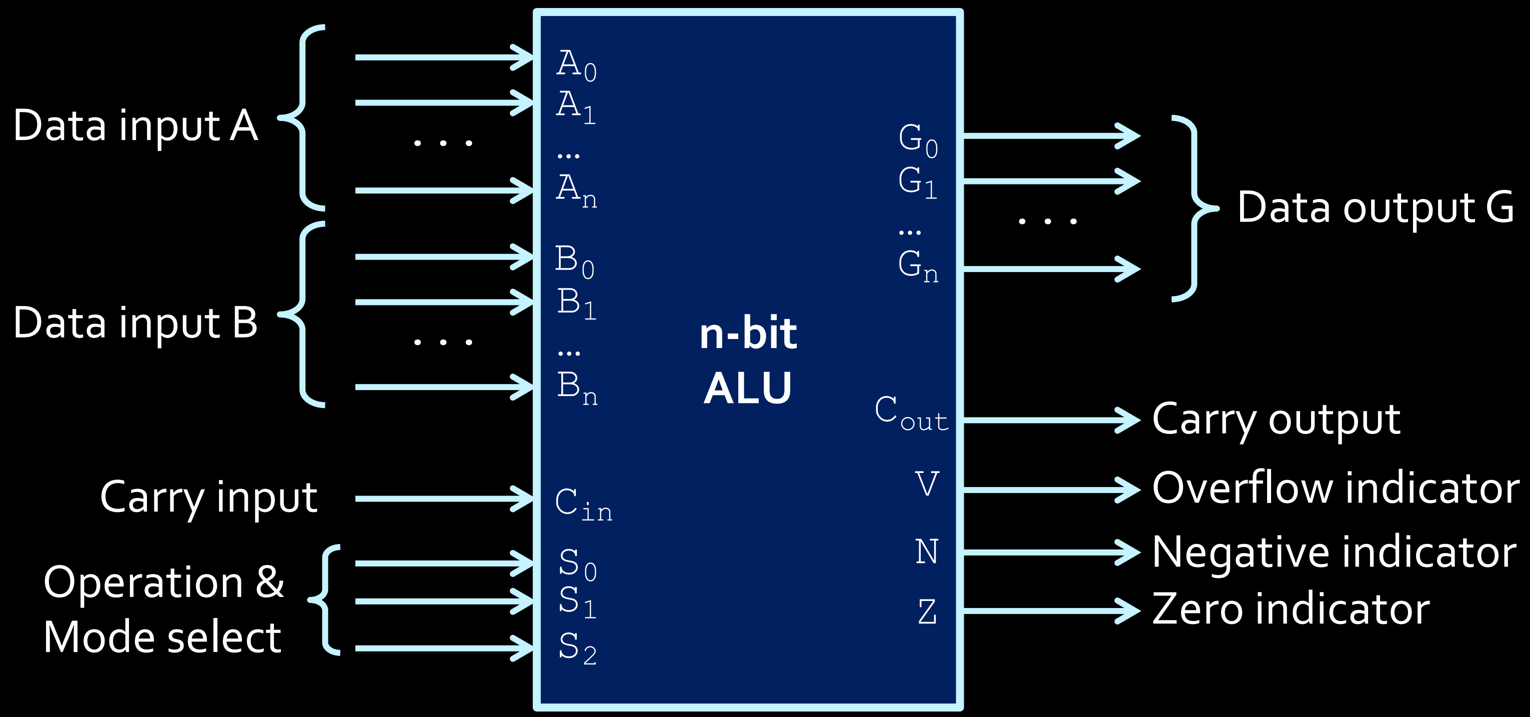
The “A” of ALU

- To understand how the ALU does all of these operations, let’s start with the arithmetic side.
- Fundamentally, this side is made of an adder / subtractor unit, which we’ve seen already:

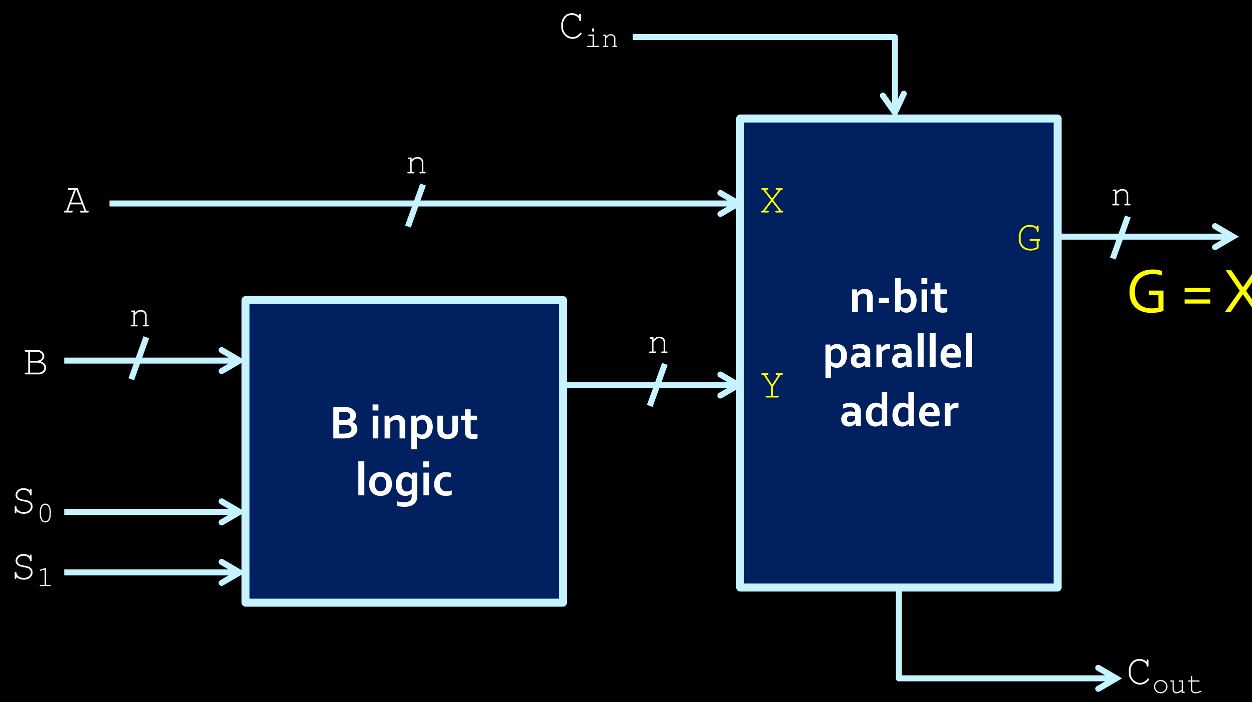


ALU block diagram

- In addition to data inputs and outputs, this circuit also has:
  - outputs indicating the different conditions,
  - inputs specifying the operation to perform (similar to `Sub`).



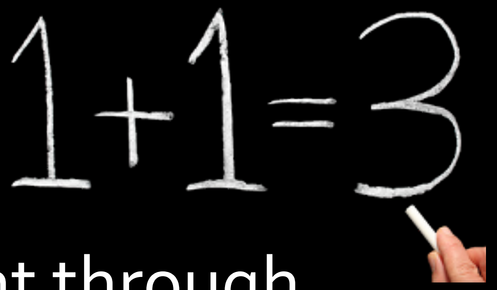
Arithmetic components



$G = X + Y + C_{in}$

- In addition to addition and subtraction, many more operations can be performed by manipulating what is added to input B, as shown in the diagram above.

Arithmetic operations



- If the input logic circuit on the left sends B straight through to the adder, result is $G = A+B$
- What if B was replaced by all ones instead?
  - Result of addition operation: $G = A-1$
- What if B was replaced by $\overline{B}$?
  - Result of addition operation: $G = A-B-1$
- And what if B was replaced by all zeroes?
  - Result is: $G = A$. (Not interesting, but useful!)

→ Instead of a Sub signal, the operation you want is signaled using the select bits S_0 & S_1.

Operation selection

Select bits		Y input	Result	Operation
S_1	S_0			
0	0	All 0s	G = A	Transfer
0	1	B	G = A+B	Addition
1	0	$\overline{B}$	G = A+$\overline{B}$	Subtraction - 1
1	1	All 1s	G = A-1	Decrement

- This is a good start! But something is missing…
- What about the carry bit?

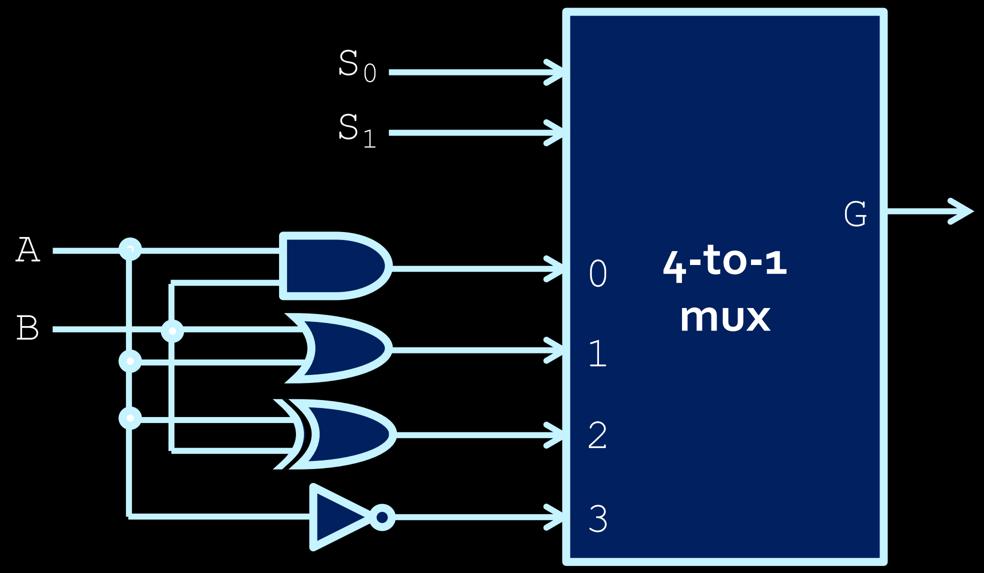
Full operation selection

Select		Input	Operation	
S_1	S_0	Y	C_{in}=0	C_{in}=1
0	0	All 0s	G = A (transfer)	G = A+1 (increment)
0	1	B	G = A+B (add)	G = A+B+1
1	0	$\overline{B}$	G = A+$\overline{B}$	G = A+$\overline{B}$+1 (subtract)
1	1	All 1s	G = A-1 (decrement)	G = A (transfer)

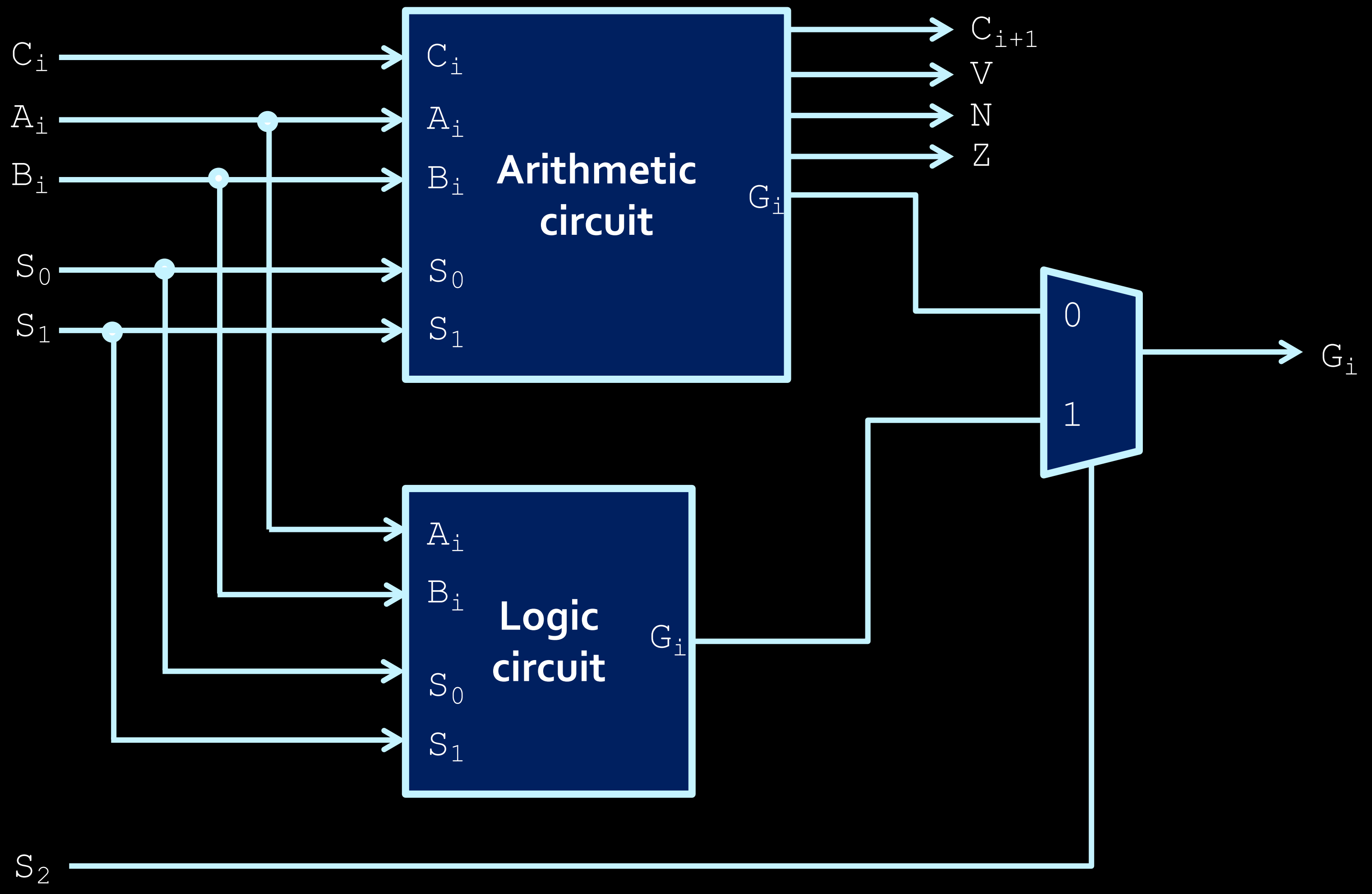
- Based on the values on the select bits and the carry bit, we can perform any number of basic arithmetic operations by manipulating what value is added to `A`.

The "L" of ALU

- We also want a circuit that can perform **logical operations**, in addition to arithmetic ones.
- How do we tell which operation to perform?
  - Another select bit!
- If $S_2 = 1$, then logic circuit block is activated.
- Multiplexer is used to determine which block (logical or arithmetic) goes to the output.



Single ALU Stage

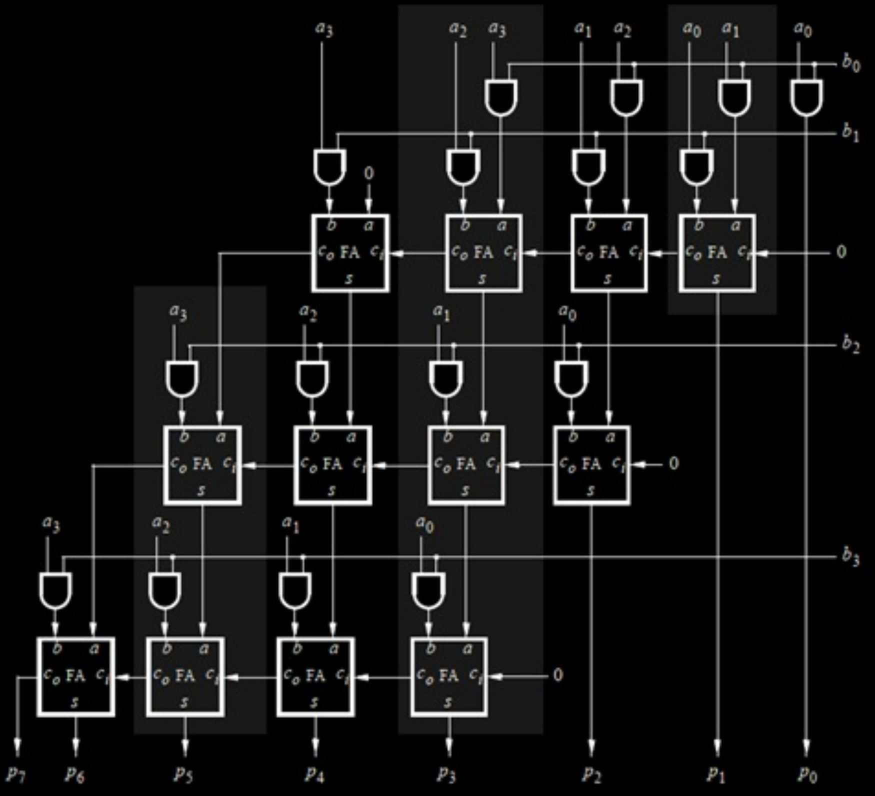


Multiplication

What about multiplication?

- Multiplication (and division) operations are always more complicated than other arithmetic (addition, subtraction) or logical (AND, OR) operations.
- Three major ways that multiplication can be implemented in circuitry:
  - Layered rows of adder units.
  - An adder/shifter circuit
  - Booth's Algorithm

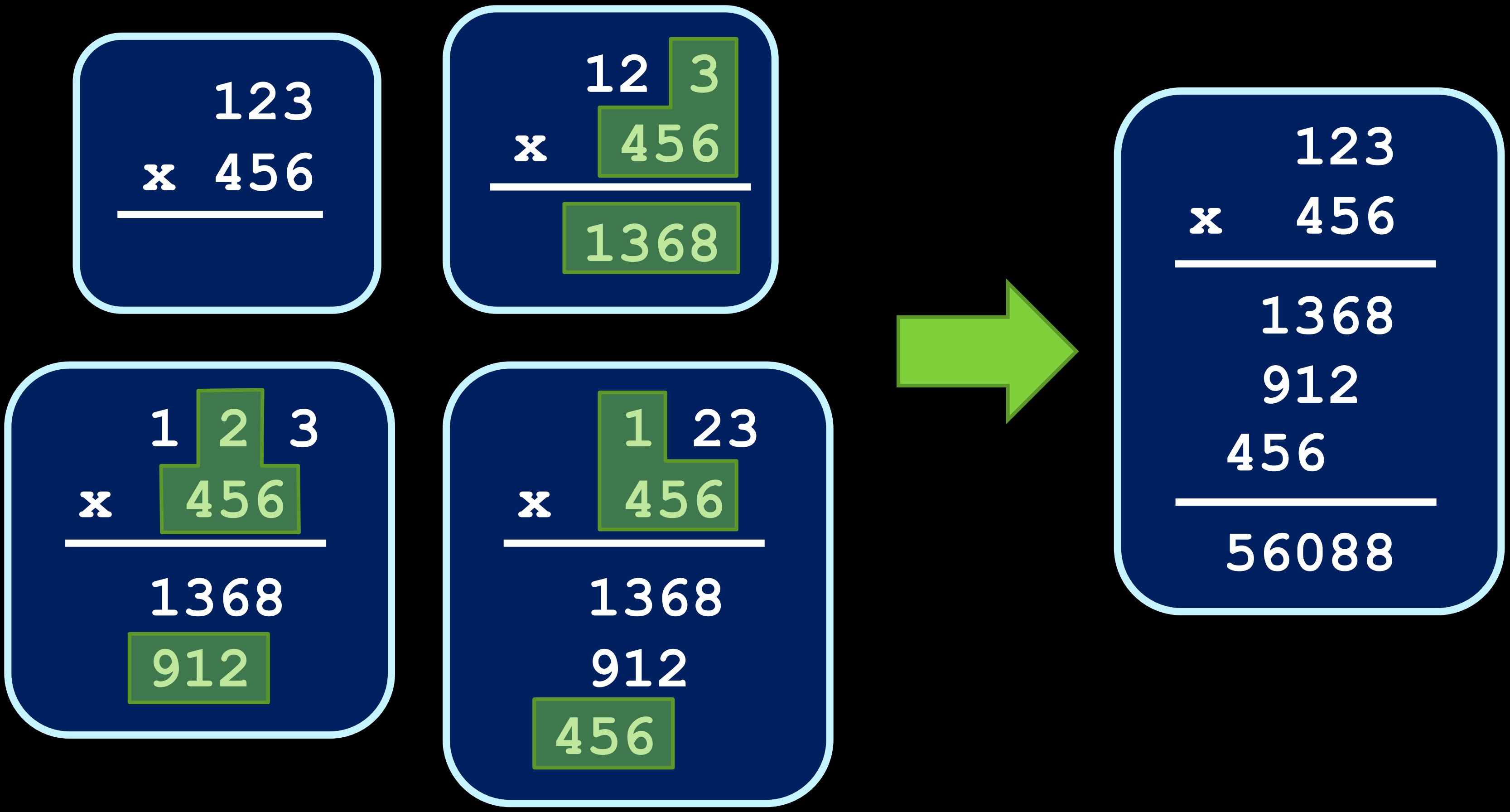
Multiplication



- Multiplier circuits can be constructed as an array of adder circuits.
- This can get a little expensive as the size of the operands grows.
- Is there an alternative to this circuit?

Multiplication

- Revisiting grade 3 math…



Multiplication

- And now, in binary...

```
  101
x 110
-----
```

```
  10 1
x  110
------
   110
```

```
  1 0 1
x   110
-------
    110
   000
```

```
  1 01
x  110
------
   110
  000
 110
```

```
    101
x   110
-------
    110
   000
  110
-------
  11110
```

Observations

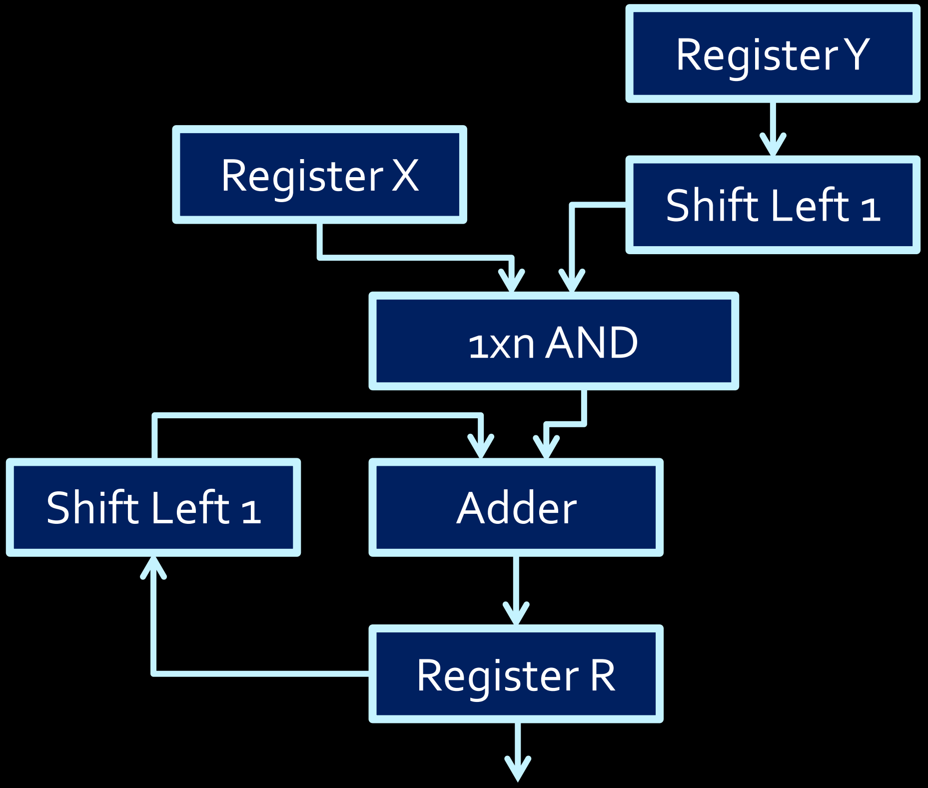
- Calculation flow
  - Multiply by 1 bit of multiplier
  - Add to sum and shift sum
  - Shift multiplier by 1 bit
  - Repeat the above
- What is "multiply by 1 bit of binary"?
  - `10101 x 1 ?`
  - `10101 x 0 ?`
  - It's an **AND**!

```
     101
x    110
--------
     110
    000
   110
--------
   11110
```

Accumulator circuits

- What if you could perform each stage of the multiplication operation, one after the other?
  - This circuit would only need a single row of adders and a couple of shift registers.

```
     101
x    110
---------
   110
    000
     110
---------
   11110
```



Make it more efficient

Think about 258 x 9999

- Multiply by 9, add to sum, shift, multiply by 9, add to sum, shift, multiple by 9, add to sum, shift, multiply by 9, add to sum.

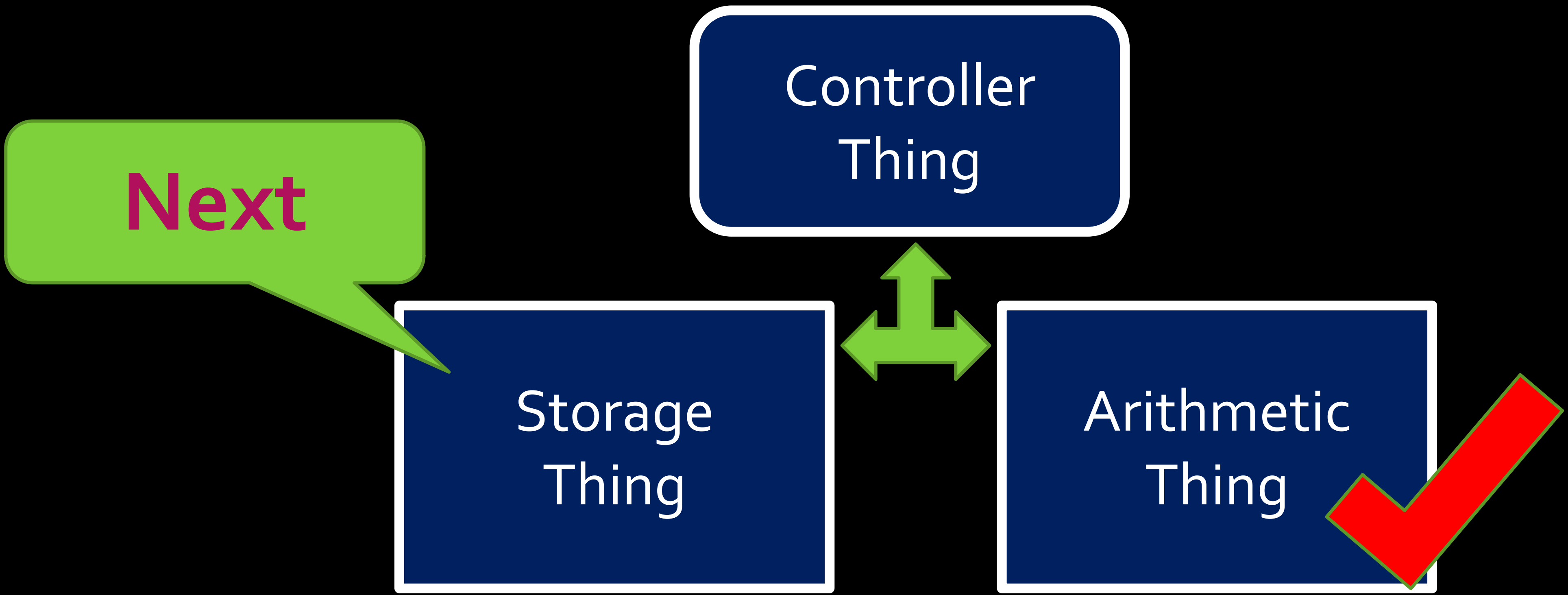
- 258 x 9999 = 258 x (10000 - 1) = 258 x 10000 – 258
- Just shift 258, becomes 2580000, then do 2580000 – 258
- More efficient!

Efficient Multiplication: Booth's Algorithm

- Take advantage of circuits where shifting is cheaper than adding, or where space is at a premium.
  - when multiplying by certain values (e.g. `99`), it can be easier to think of this operation as a difference between two products.
- Consider the shortcut method when multiplying a given decimal value `X` by `9999`:
  - `X*9999 = X*10000 - X*1`
- Now consider the equivalent problem in binary:
  - `X*001111 = X*010000 - X*1`
- More details: https://en.wikipedia.org/wiki/Booth%27s_multiplication_algorithm

Reflections on multiplication

- Multiplication isn't as common an operation as addition or subtraction, but occurs enough that its implementation is handled in the hardware.
- Most common multiplication and division operations are powers of 2. For this, we do shifting instead of using the multiplier circuit.
  - e.g., in your code, do `x << 3`, instead of `x * 8`


Next
Controller Thing
Storage Thing
Arithmetic Thing