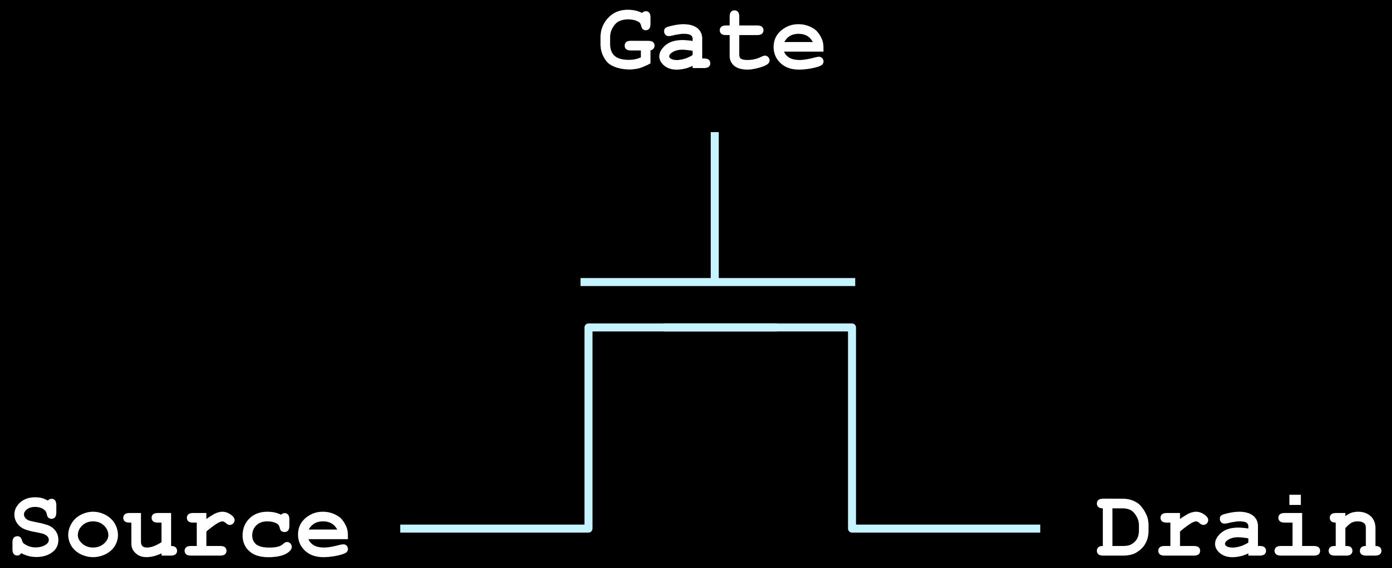
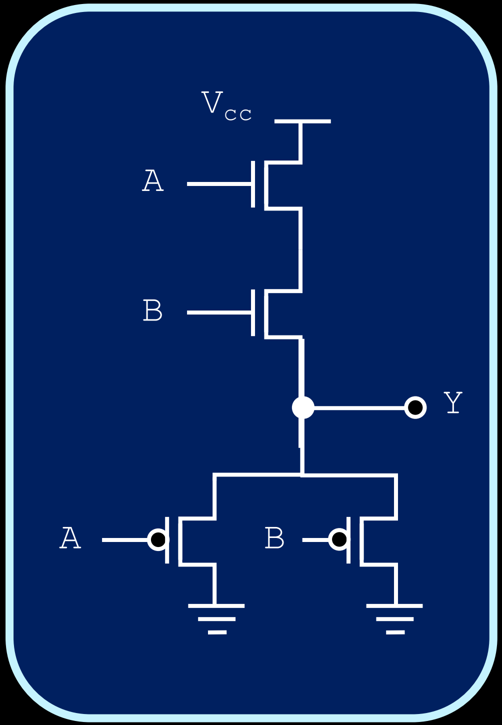
CSC258 Week 2

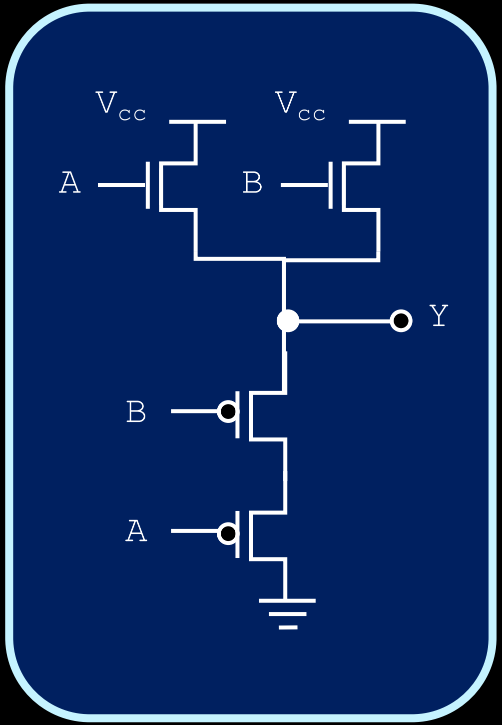
Recap: MOSFET



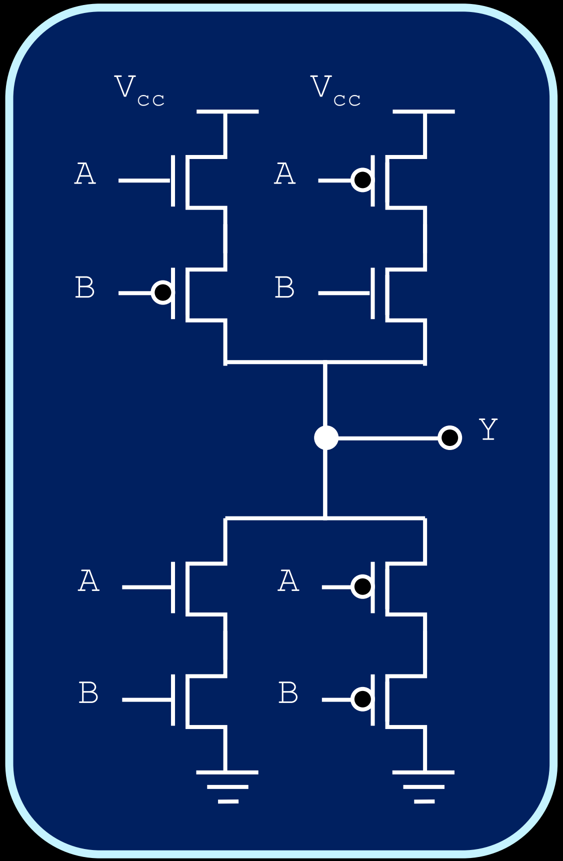
Recap: Transistors into logic gates



AND

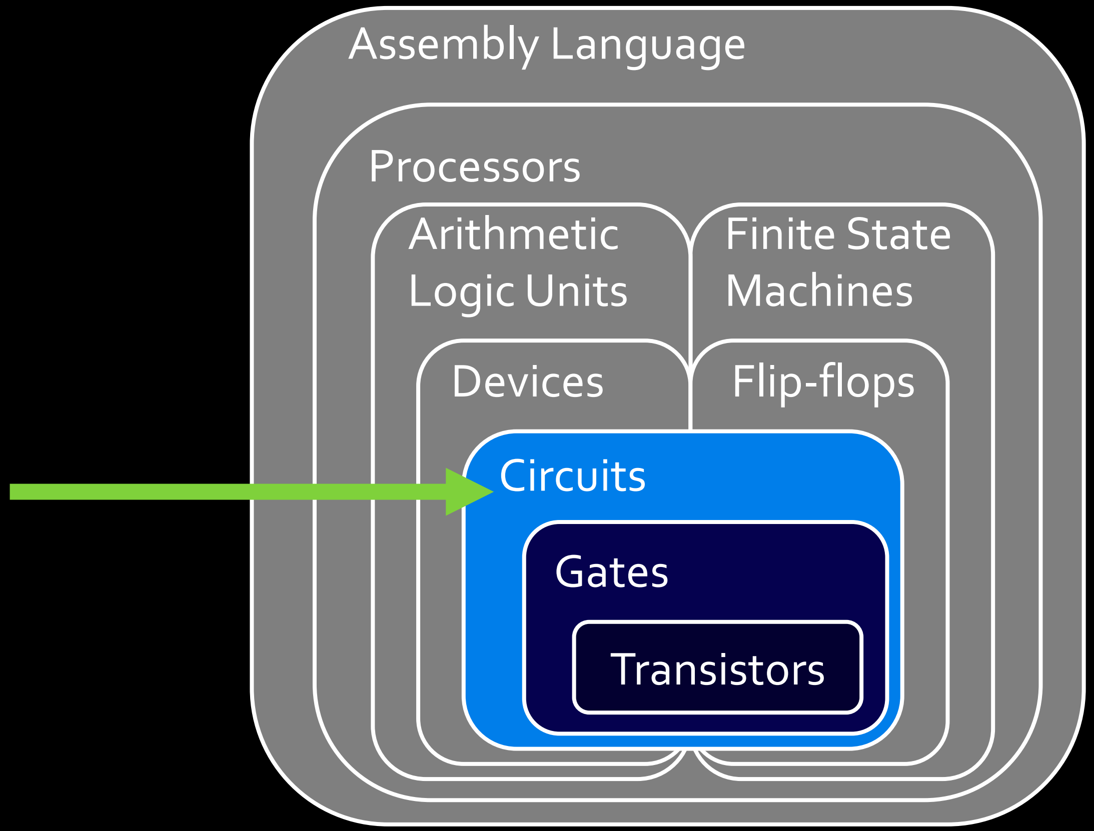


OR



XOR

Next: From gates to circuits



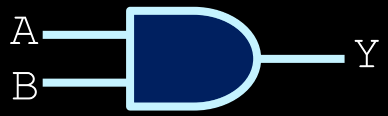
The goal

- Use the gates as building blocks to build large circuits that represent complex logics.

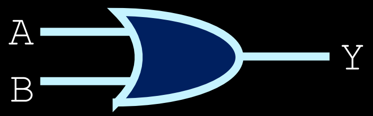
- The beauty of abstraction in system design: from this point on, we will just use the symbolic logic gates (AND, OR, XOR, etc) without having to think about the lower-level details (MOSFET, pn-junctions, etc).

Making logic with gates

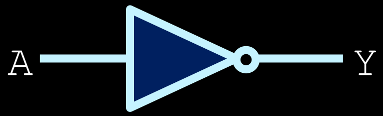
- Logic gates like the following allow us to create an output value, based on one or more input values.
  - Each corresponds to Boolean logic that we've seen before in CSC108 and CSC148:



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



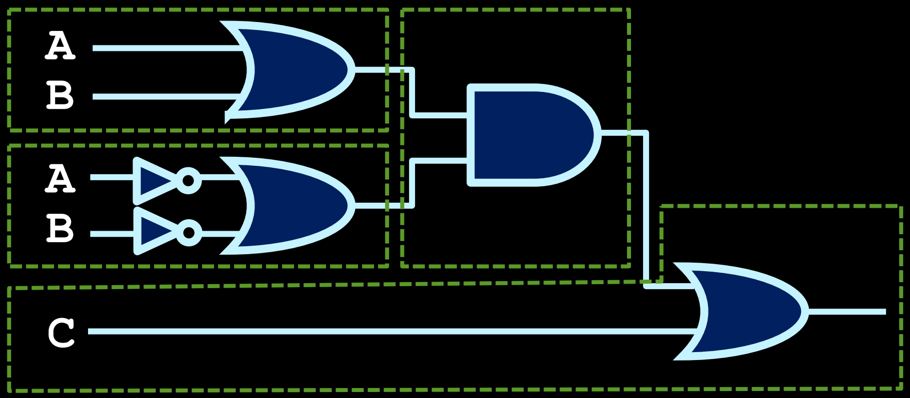
A	Y
0	1
1	0

Making Boolean expressions

- So how would you represent Boolean expressions using logic gates?

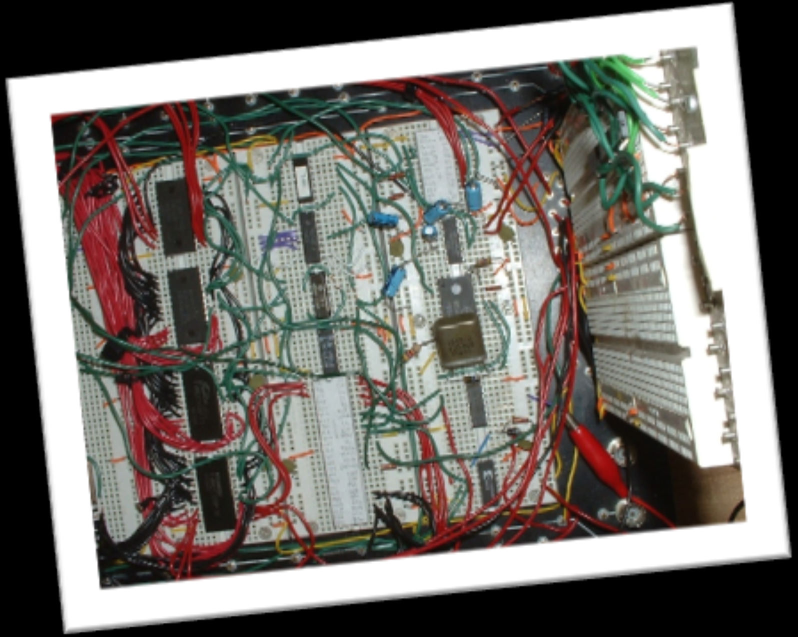
```
Y = (A or B) and (not A or not B) or C
```

- Like so:



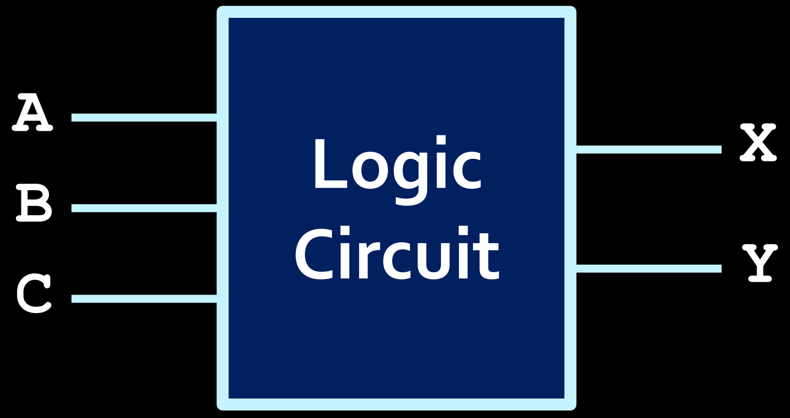
Creating complex circuits

- What do we do in the case of more complex circuits, with several inputs and more than one output?
  - If you're lucky, a truth table is provided to express the circuit.
  - Usually, the behaviour of the circuit is expressed in words, and the first step involves creating a truth table that represents the described behaviour.



Circuit example

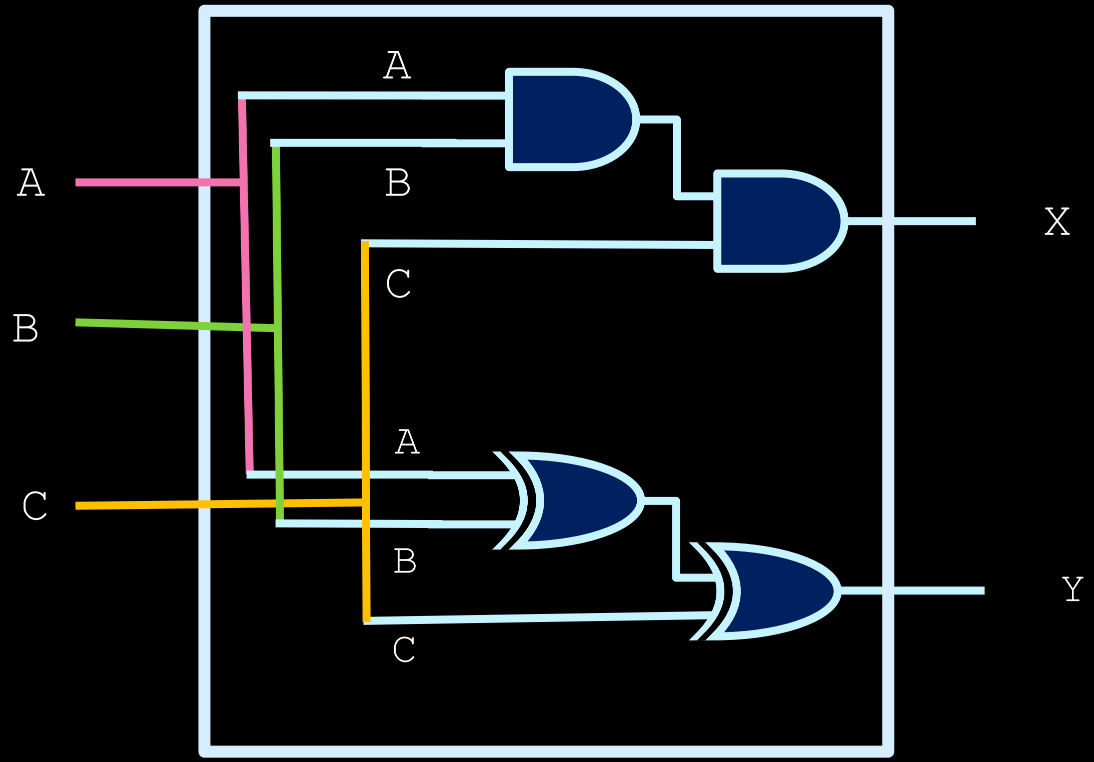
- The circuit on the right has three inputs (A, B and C) and two outputs (X and Y).



- What logic is needed to set X high when all three inputs are high?
- What logic is needed to set Y high when the number of high inputs is odd?

Combinational circuits

- Small problems can be solved easily.



X high when all three inputs are high

Y high when number of high is odd

For more complicated circuits,
we need a systematical approach

Creating complex logic

- The general approach
- Basic steps:
  1. Create truth tables based on the desired behaviour of the circuit.
  2. Come up with a "good" Boolean expression that has exactly that truth table.
  3. Convert Boolean expression to gates.
- The key to an efficient design?
  - Spending extra time on Step #2.

First,
a better way to represent
truth tables

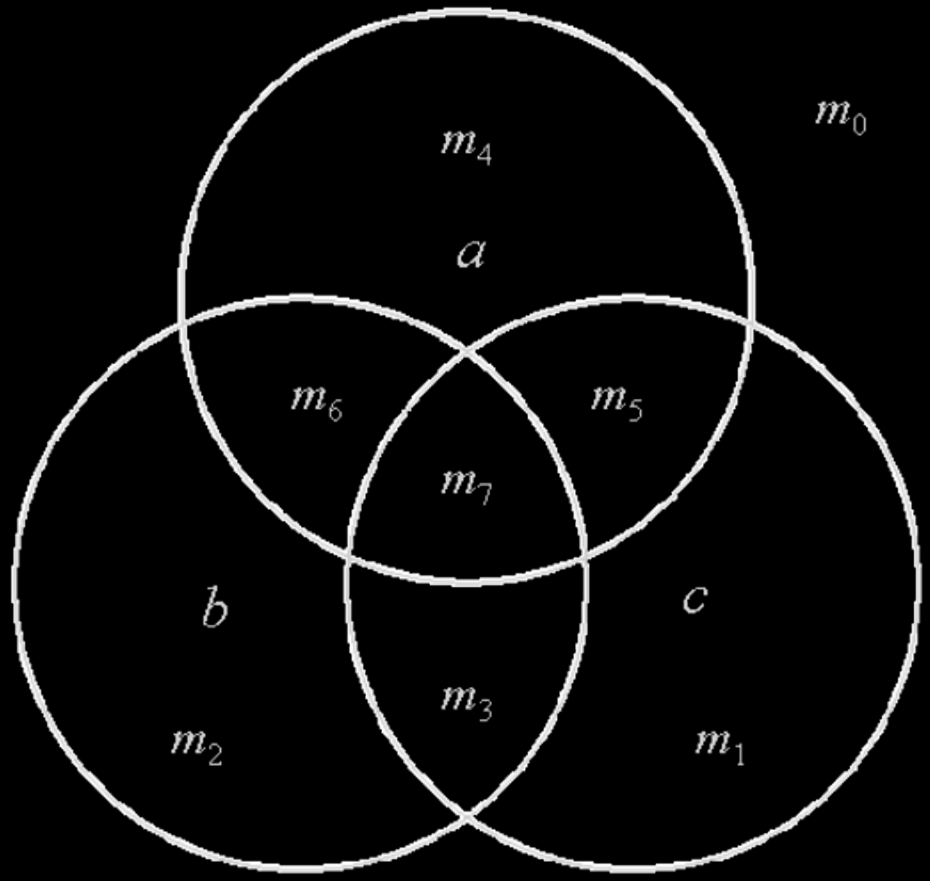
Example truth table

- Consider the following example:
  - *"Y is high only when B and C are both high"*
- This leads to the truth table on the right.
  - Do we always have to draw the whole table?
  - Is there a better way to describe the truth table?

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

This is all we needed to express!

Yes, use "Minterms" and "Maxterms"



Warm-Up Exercise

For each of the following logic expressions, what are the A, B, C values that make the expression evaluate to 1 ?

A'B'C'

- A=0, B=0, C=0, and only this

ABC

- 111 and only this

A'BC

- 011 and only this

ABC'

- 110 and only this

Minterms, informally

- First, sort the rows according to the value of the number “ABC” represents
- Then for each row, find the **AND** expression that evaluates to 1 iff ABC are of the values in the row. We name the AND expression as $m_{\{row\ number\}}$

Sorted

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

A’B’C’	m_0
A’B’C	m_1
A’BC’	m_2
A’BC	m_3
AB’C’	m_4
AB’C	m_5
ABC’	m_6
ABC	m_7

Minterm, a more formal description

Minterm: an AND expression with every input present in true or complemented form.

$m_0: \overline{A} \cdot \overline{B} \cdot \overline{C}$

$m_1: \overline{A} \cdot \overline{B} \cdot C$

$m_2: \overline{A} \cdot B \cdot \overline{C}$

$m_3: \overline{A} \cdot B \cdot C$

$m_7: A \cdot B \cdot C$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Minterm	Y
m_0	0
m_1	1
m_2	1
m_3	1
m_4	1
m_5	0
m_6	1
m_7	0

Minterm (m) and Maxterm (M)

Minterm: an **AND** expression with every input present in true or complemented form.

Maxterm: an **OR** expression with every input present in true or complemented form.

M_0: $A+B+C$ M_1: $A+B+\overline{C}$

M_6: $\overline{A}+\overline{B}+C$ M_7: $\overline{A}+\overline{B}+\overline{C}$

Feel something fishy?

Naming!

Minterm is about when the output is 1

m_0 is $\overline{A} \cdot \overline{B} \cdot \overline{C}$

$\overline{A} \cdot \overline{B} \cdot \overline{C}$ is **1** only when A, B, C are 0, 0, 0

M_0 is $A+B+C$

$A+B+C$ is **0** only when A, B, C are 0, 0, 0

Maxterm is about when the output is 0

Exercise: Minterm or Maxterm?

given four inputs: (A, B, C, D)

$A \cdot B \cdot C$ — Neither! **Every** input needs to be there, D is missing!

$A+B+D$ — Neither! Same reason

$A \cdot B + C \cdot \overline{D}$ — Neither! It has to be **only AND** or **only OR**, cannot mix

$A+\overline{B}+C+\overline{D}$ — Maxterm, M_5

$A \cdot \overline{B} \cdot C \cdot \overline{D}$ — Minterm, m_{10}

Quick fact

- Given n inputs, how many possible minterms and maxterms are there?
  - 2^n minterms and 2^n maxterms possible (same as the number of rows in a truth table).

Quick note about notations

- AND operations are denoted in these expressions by the multiplication symbol.
  - e.g. $A \cdot B \cdot C$ or $A*B*C$ or $A \wedge B \wedge C$
- OR operations are denoted by the addition symbol.
  - e.g. $A+B+C$ or $A \vee B \vee C$
- NOT is denoted by multiple symbols.
  - e.g. $\neg A$ or A' or $\overline{A}$
- XOR occurs rarely in circuit expressions.
  - e.g. $A \oplus B$

Use minterms and maxterms

to go from truth table to logic expression

Using minterms

- What are minterms used for?
  - A single minterm indicates a set of inputs that will make the output go high.
  - Example: Describe the truth table on the right using minterm:
    - m_2 A'B'CD'

A	B	C	D	m_2
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Using minterms

- What happens when you OR two minterms?
  - Result is output that goes high in both minterm cases.
  - Describe the truth table with the right-most column of outputs
  - **$m_2 + m_8$**

A	B	C	D	m_2	m_8	m_2+m_8
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	0	1
0	0	1	1	0	0	0
0	1	0	0	0	0	0
0	1	0	1	0	0	0
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	1	1
1	0	0	1	0	0	0
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	0	0	0

We came up with a logic expression that has the desired truth table, easily: A'B'CD' + AB'C'D'

Creating Boolean expressions

- Two canonical forms of boolean expressions:
  - Sum-of-Minterms (SOM): AB + A'B + AB'
    - Each minterm corresponds to a single high output in the truth table.
    - Also known as: Sum-of-Products.
  - Product-of-Maxterms (POM): (A+B)(A' + B)(A+B')
    - Each maxterm corresponds to a single low output in the truth table.
    - Also known as Product-of-Sums.

Every logic expression can be converted to a SOM, also to a POM.

$$Y = m_2 + m_6 + m_7 + m_{10} \quad \text{(SOM)}$$

A	B	C	D	m_2	m_6	m_7	m_{10}	Y
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0

$Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14}$ (POM)

A	B	C	D	M_3	M_5	M_7	M_{10}	M_{14}	Y
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	0
0	1	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	0
1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	1
1	0	1	0	1	1	1	0	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	0	0
1	1	1	1	1	1	1	1	1	1

Sum-of-Minterms vs Product-of-Maxterm

- SOM expresses which inputs cause the output to go high.
- POM expresses which inputs cause the output to go low
- SOMs are useful in cases with very few input combinations that produce high output.
- POMs are useful when expressing truth tables that have very few low output cases…

What if we do this using POM?

$$Y = m_2 + m_6 + m_7 + m_{10} \quad (SOM)$$

A	B	C	D	m_2	m_6	m_7	m_{10}	Y
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	1	0	0	1
0	1	1	1	0	0	1	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0

What if we do this using SOM?

$$Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14} \text{ (POM)}$$

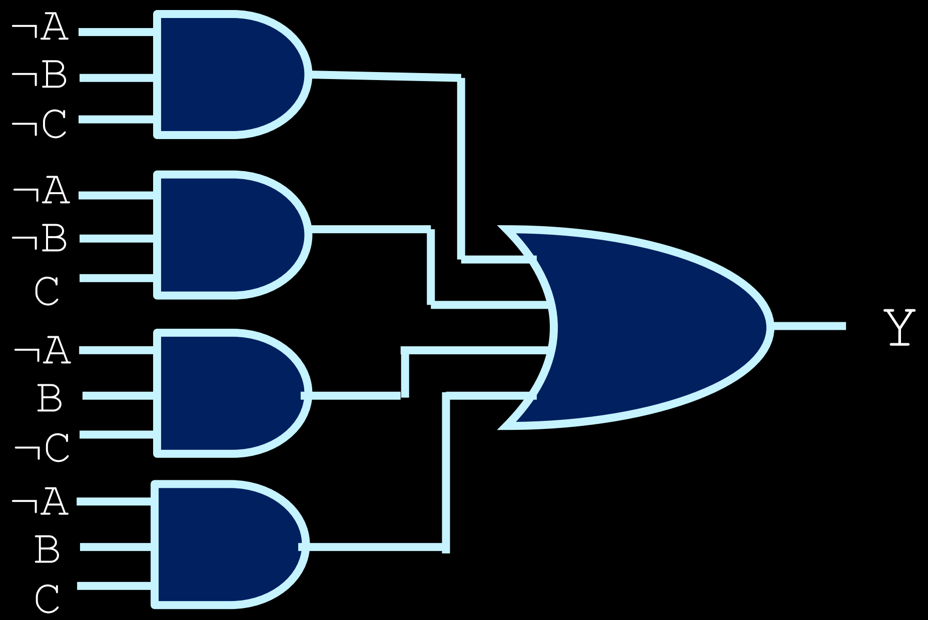
A	B	C	D	M_3	M_5	M_7	M_{10}	M_{14}	Y
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	0
0	1	0	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	0
0	1	1	0	1	1	1	1	1	1
0	1	1	1	1	1	0	1	1	0
1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	1
1	0	1	0	1	1	1	0	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	0	0
1	1	1	1	1	1	1	1	1	1

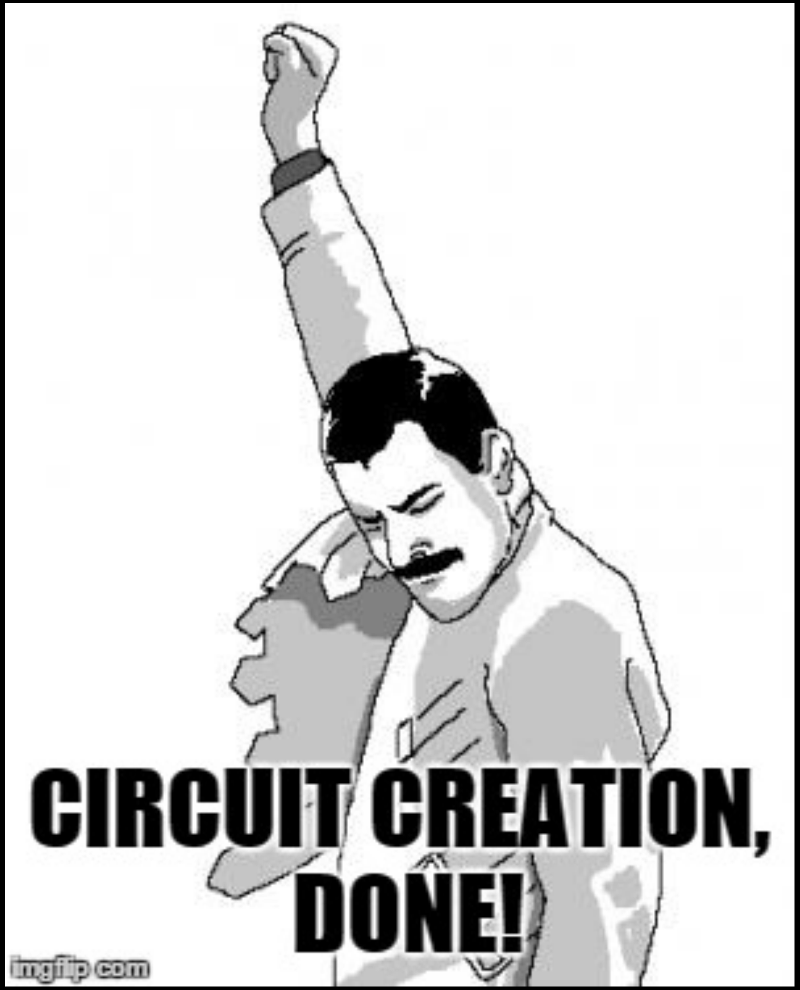
Converting SOM to gates

- Once you have a Sum-of-Minterms expression, it is easy to convert this to the equivalent combination of gates:

$$m_0 + m_1 + m_2 + m_3 =$$

$$\overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot\overline{C} + \overline{A}\cdot B\cdot C$$




CIRCUIT CREATION,
DONE!
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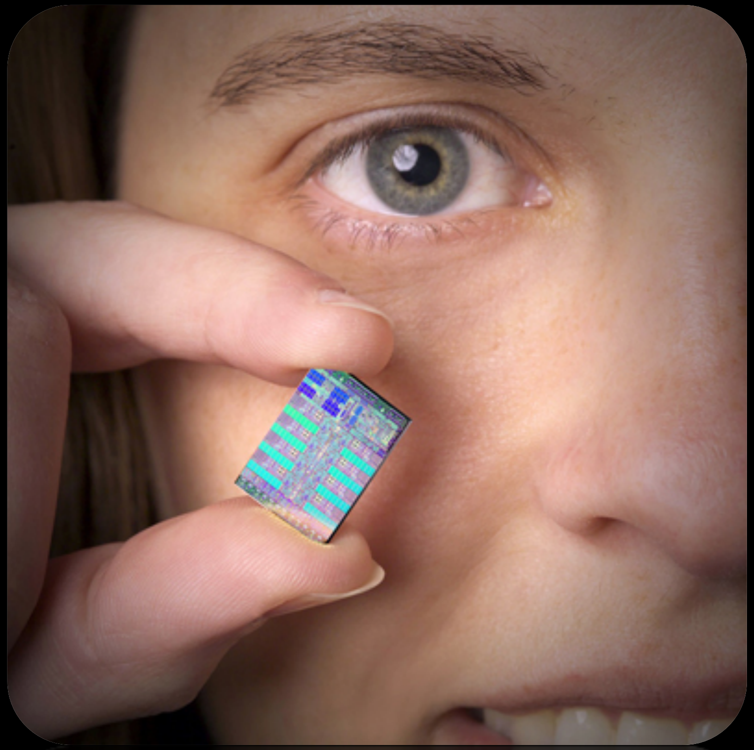
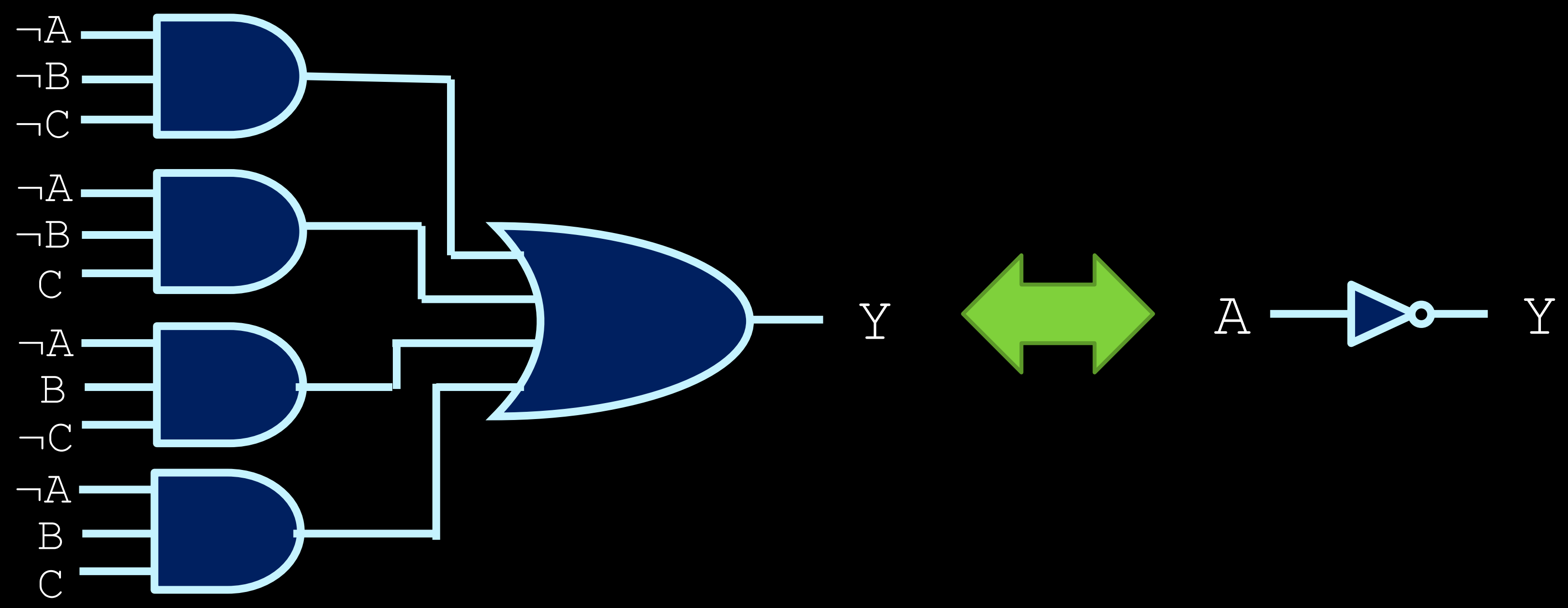
NO
ITS NOT
memegenerator.net

Reducing circuits



Reasons for reducing circuits



- To minimize the number of gates, we want to reduce the Boolean expression as much as possible from a collection of minterms to something smaller.
- This is where math skills come in handy ☺

Boolean algebra review

- Axioms:

$$0 \cdot 0 = 0 \qquad 0 \cdot 1 = 1 \cdot 0 = 0$$
$$1 \cdot 1 = 1 \qquad \text{if } x = 1,\ \overline{x} = 0$$

- From this, we can extrapolate:

$$x \cdot 0 = 0 \qquad x+1 = 1$$
$$x \cdot 1 = x \qquad x+0 = x$$
$$x \cdot x = x \qquad x+x = x$$
$$x \cdot \overline{x} = 0 \qquad x+\overline{x} = 1$$
$$\overline{\overline{x}} = x$$

Other boolean identities

- Commutative Law:

$$x \cdot y = y \cdot x \qquad x + y = y + x$$

- Associative Law:

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$
$$x + (y + z) = (x + y) + z$$

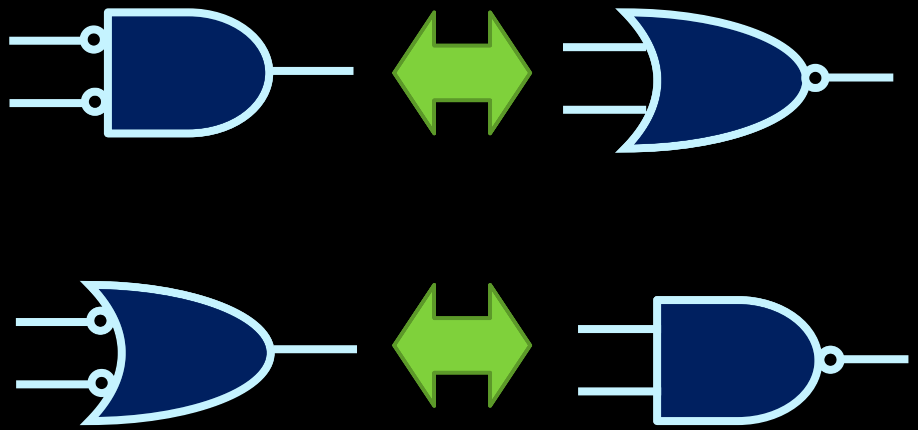
- Distributive Law:

$$x \cdot (y + z) = x \cdot y + x \cdot z$$
$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

Other boolean identities

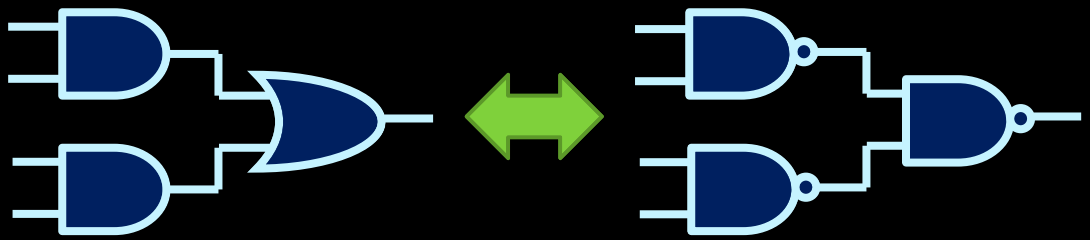
- De Morgan's Laws:

$$\overline{x} \cdot \overline{y} = \overline{x+y}$$
$$\overline{x} + \overline{y} = \overline{x \cdot y}$$

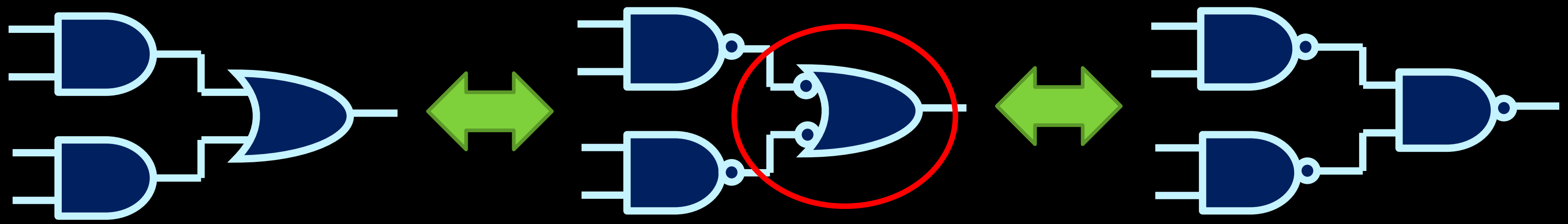


De Morgan and NAND gates

- De Morgan's Law is important because out of all the gates, NANDs are the cheapest to fabricate.
  - a Sum-of-Products circuit could be converted into an equivalent circuit of NAND gates:



- This is all based on de Morgan's Law:



Reducing Boolean expressions

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

- Assuming logic specs at left, we get the following:

$$m_3 + m_4 + m_6 + m_7$$

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

Warming up…

$$A \cdot B + A \cdot \overline{B} = A$$

Reduce by combing two terms that differ by a single literal.

Let’s reduce this

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

Combine the last two terms…

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B$$

Combine the middle two and the end two …

$$Y = B \cdot C + A \cdot \overline{C}$$

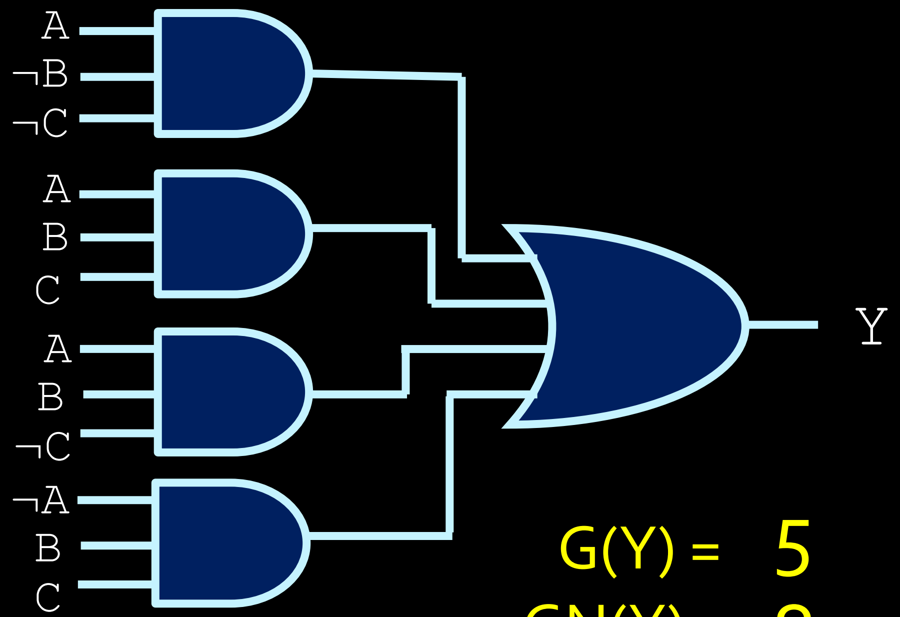
There could be different ways of combining, some are **simpler** than others.

How to get to the simplest expression?

Wait … What does “simplest” mean?

What is "simplest"?

- In this case, "simple" denotes the lowest **gate cost (G)** or the lowest **gate cost with NOTs (GN)**.
- To calculate the gate cost, simply add all the gates together (as well as the cost of the NOT gates, in the case of the GN cost).

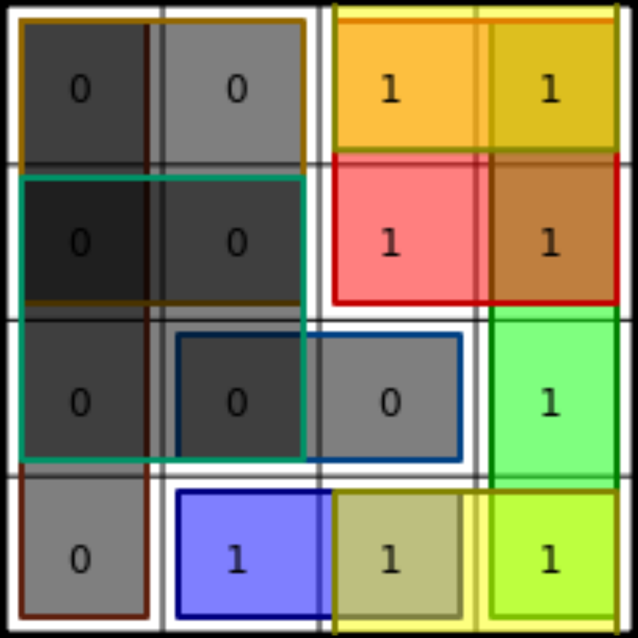


G(Y) = 5
GN(Y) = 8

Don't count ¬C twice!

Karnaugh maps

Find the simplest expression, systematically.



Reducing Boolean expressions

- How do we find the "simplest" expression for a circuit?
  - Technique called Karnaugh maps (or K-maps).
  - Karnaugh maps are a 2D grid of minterms, where adjacent minterm locations in the grid differ by a single literal.
  - Values of the grid are the output for that minterm.

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

Compare these…

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

$$Y = \overline{A}\cdot B\cdot C + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot\overline{C} + A\cdot B\cdot C$$

Karnaugh maps

- Karnaugh maps can be of any size and have any number of inputs.
- Since adjacent minterms only differ by a single literal, they can be combined into a single term that omits that value.

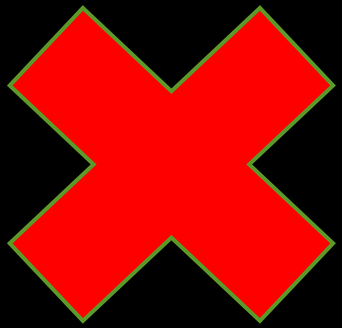
	$\overline{C} \cdot \overline{D}$	$\overline{C} \cdot D$	$C \cdot D$	$C \cdot \overline{D}$
$\overline{A} \cdot \overline{B}$	m_0	m_1	m_3	m_2
$\overline{A} \cdot B$	m_4	m_5	m_7	m_6
$A \cdot B$	m_{12}	m_{13}	m_{15}	m_{14}
$A \cdot \overline{B}$	m_8	m_9	m_{11}	m_{10}

Using Karnaugh maps

- Once Karnaugh maps are created, draw boxes over groups of high output values.
  - Boxes must be rectangular, and aligned with map.
  - Number of values contained within each box must be a power of 2.
  - Boxes may overlap with each other.
  - Boxes may wrap across edges of map.

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	1	1	0
A	0	0	1	0

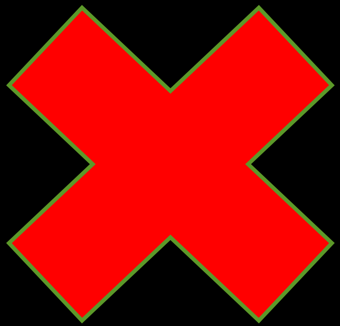


Must be rectangle!

	$\bar{B} \cdot \bar{C}$	$\bar{B} \cdot C$	$B \cdot C$	$B \cdot \bar{C}$
$\bar{A}$	0	1	1	0
A	0	0	1	0

Two boxes overlapping each other is fine.

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	1	1	1
A	0	0	0	0



Number of value contained must be power of 2.

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	1	1	1
A	0	0	0	0

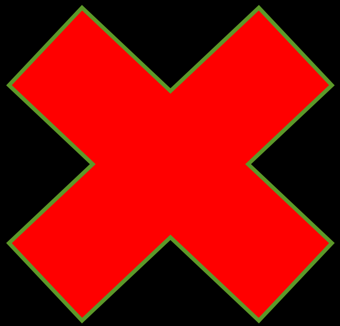
1 is a power of 2

$1 = 2^0$

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	1	1	0
A	0	1	1	0

Rectangle, with power of 2 entries

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	1	0	0
A	0	0	1	0



Must be aligned with map.

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	0	0
A	1	0	0	1

Wrapping across edge is fine.

So… how to find the smallest expression

Minterms in one box can be combined into one term

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	1	0
A	0	0	1	0

$$\overline{A} \cdot B \cdot C + A \cdot B \cdot C = B \cdot C$$

So… how to find smallest expression

The simplest expression corresponds to the **smallest number of boxes** that cover all the high values (1's).

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

So… how to find smallest expression

And each box should be as **large** as possible.

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	1	1	1
A	0	0	1	1

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	1	1	1
A	0	0	1	1

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B$$

	$\overline{B} \cdot \overline{C}$	$\overline{B} \cdot C$	$B \cdot C$	$B \cdot \overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

$$Y = B \cdot C + A \cdot \overline{C}$$

K-map: the steps

Given a complicated expression

1. Convert it to Sum-Of-Minterms
2. Draw the 2D grid
3. Mark all the high values (1's), according to which minterms are in the SOM.
4. Draw boxes that cover 1's.
5. Find the smallest set of boxes that cover all 1's.
6. Write out the simpfied result according to the boxes found.

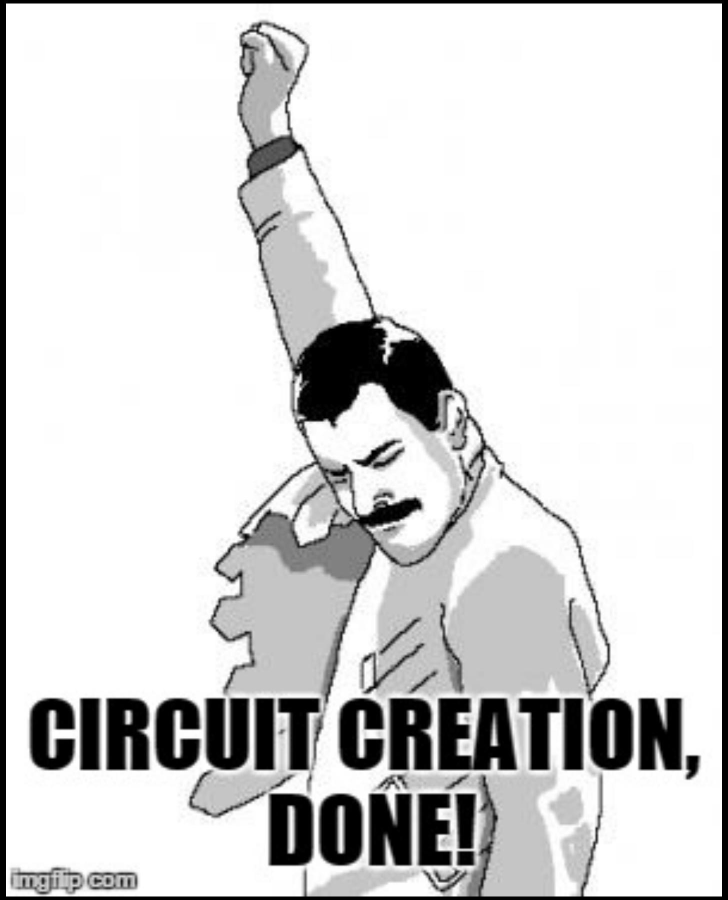
Everything can be done using Maxterms, too

- Can also use this technique to group maxterms together as well.
- Karnaugh maps with maxterms involves grouping the zero entries together, instead of grouping the entries with one values.

	C+D	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
A+B	M_0	M_1	M_3	M_2
$A+\overline{B}$	M_4	M_5	M_7	M_6
$\overline{A}+\overline{B}$	M_{12}	M_{13}	M_{15}	M_{14}
$\overline{A}+B$	M_8	M_9	M_{11}	M_{10}

Circuit creation – the whole flow

1. Understand desired behaviour
2. Write the truth table based on the behaviour
3. Write the SOM (or POM) of that truth table
4. Simplify the SOM using K-map
5. Translate the simplified logic expression into circuit with gates.



Today we learned

- How to create a logic circuit from scratch, given a desired digital behaviour.
- Minterm & Maxterm
- K-Map use to reduce the circuit

Next Week:

- Logical Devices